MATH 54 – HINTS TO HOMEWORK 16

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Here are a couple of hints to Homework 16. Enjoy!

SECTION 6.2: ORTHOGONAL SETS

Remember: A set $\mathcal{B}$ is orthogonal if for every pair of distinct vectors $\mathbf{u}$ and $\mathbf{v}$, $\mathbf{u} \cdot \mathbf{v} = 0$. It is orthonormal if it is orthogonal and every vector has length 1. An orthogonal set can be made orthonormal by dividing every vector by its length.

6.2.7.

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} \frac{\mathbf{x}\cdot\mathbf{u_1}}{\mathbf{u_1}\cdot\mathbf{u_1}} \\ \frac{\mathbf{x}\cdot\mathbf{u_2}}{\mathbf{u_2}\cdot\mathbf{u_2}} \end{bmatrix}$$

6.2.9.

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} \frac{\mathbf{x}\cdot\mathbf{u_1}}{\mathbf{u_1}\cdot\mathbf{u_1}} \\ \frac{\mathbf{x}\cdot\mathbf{u_2}}{\mathbf{u_2}\cdot\mathbf{u_2}} \\ \frac{\mathbf{x}\cdot\mathbf{u_3}}{\mathbf{u_3}\cdot\mathbf{u_3}} \end{bmatrix}$$

6.2.13, 6.2.15. The formula for orthogonal projection of $\mathbf{y}$ on the line spanned by $\mathbf{u}$ is:

$$\hat{\mathbf{y}} = \left(\frac{\mathbf{y}\cdot\mathbf{u}}{\mathbf{u}\cdot\mathbf{u}}\right)\mathbf{u}$$

Then you can write $\mathbf{y} = \hat{\mathbf{y}} + (\mathbf{y} - \hat{\mathbf{y}})$ Notice that $\hat{\mathbf{y}}$ is in the span of $\mathbf{u}$, whereas $(\mathbf{y} - \hat{\mathbf{y}})$ is orthogonal to $\mathbf{u}$.

The distance between $\mathbf{u}$ and L is then $\|\mathbf{y} - \hat{\mathbf{y}}\|$

6.2.23.

(a) **T** (Take $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ in $\mathbb{R}^3$)

(b) **T** (You just use the formula in Theorem 5 on page 285; THIS is what makes orthogonal sets so awesome!)

(c) **F** (They're still orthogonal, this is because $\frac{u}{\|u\|} \cdot \frac{v}{\|v\|} = \frac{1}{\|u\|\|v\|} u \cdot v = 0$ if u and v are orthogonal)

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(d) **F** (in this course, we assume that orthogonal matrices must be **square**[1])
(e) **F** (it's $\|\mathbf{y} - \hat{\mathbf{y}}\|$ which gives that distance)

SECTION 6.3: ORTHOGONAL PROJECTION

Here are all the basic facts that you'll need:

(1) If $W = Span\{\mathbf{u_1}, \mathbf{u_2} \cdots \mathbf{u_k}\}$, then the orthogonal projection of $\mathbf{y}$ onto W is:

$$\hat{\mathbf{y}} = \left(\frac{\mathbf{y} \cdot \mathbf{u_1}}{\mathbf{u_1} \cdot \mathbf{u_1}}\right)\mathbf{u_1} + \left(\frac{\mathbf{y} \cdot \mathbf{u_2}}{\mathbf{u_2} \cdot \mathbf{u_2}}\right)\mathbf{u_2} + \cdots + \left(\frac{\mathbf{y} \cdot \mathbf{u_k}}{\mathbf{u_k} \cdot \mathbf{u_k}}\right)\mathbf{u_k}$$

(2) Then $\hat{\mathbf{y}}$ is in W, $y - \hat{\mathbf{y}}$ is in $W^{\perp}$ (that is, orthogonal to W).
(3) $\mathbf{y} = (\hat{\mathbf{y}}) + (y - \hat{\mathbf{y}})$, which decomposes $\mathbf{y}$ as a sum of two vectors, one in W and the other one orthogonal to W.
(4) $\hat{\mathbf{y}}$ is the closest point to $\mathbf{y}$ in W.
(5) $\|y - \hat{\mathbf{y}}\|$ is the smallest distance between $\mathbf{y}$ and W.

6.3.21.

(a) **T**
(b) **T**
(c) **F**
(d) **T**
(e) **T**

[1]but not in other courses, beware!