MATH 54 – SOLUTION TO 6.1.24

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$$\|\mathbf{u}+\mathbf{v}\|^2 + \|\mathbf{u}-\mathbf{v}\|^2$$

$$= (\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}+\mathbf{v}) + (\mathbf{u}-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})$$

(by the definition of $\|\mathbf{x}\|^2 = \mathbf{x}\cdot\mathbf{x}$)

$$= \mathbf{u}\cdot(\mathbf{u}+\mathbf{v}) + \mathbf{v}\cdot(\mathbf{u}+\mathbf{v}) + \mathbf{u}\cdot(\mathbf{u}-\mathbf{v}) + (-\mathbf{v})\cdot(\mathbf{u}-\mathbf{v})$$

(because $\mathbf{x}\cdot(\mathbf{y}+\mathbf{z}) = \mathbf{x}\cdot\mathbf{y} + \mathbf{x}\cdot\mathbf{z}$ for all $\mathbf{x},\mathbf{y},\mathbf{z}$)

$$= \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot(-\mathbf{v}) + (-\mathbf{v})\cdot\mathbf{u} + (-\mathbf{v})\cdot(-\mathbf{v})$$

(because $\mathbf{x}\cdot(\mathbf{y}+\mathbf{z}) = \mathbf{x}\cdot\mathbf{y} + \mathbf{x}\cdot\mathbf{z}$ and $\mathbf{y}\cdot\mathbf{x} = \mathbf{x}\cdot\mathbf{y}$, for all $\mathbf{x},\mathbf{y},\mathbf{z}$)

$$= \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{v}$$

(because $(c\mathbf{x})\cdot\mathbf{y} = \mathbf{x}\cdot(c\mathbf{y}) = c\,(\mathbf{x}\cdot\mathbf{y})$ for all $\mathbf{x},\mathbf{y}$)

$$= \mathbf{u}\cdot\mathbf{u} + \mathbf{u}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v} + \mathbf{u}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} - \mathbf{u}\cdot\mathbf{v} + \mathbf{v}\cdot\mathbf{v}$$

(because $\mathbf{y}\cdot\mathbf{x} = \mathbf{x}\cdot\mathbf{y}$, for all $\mathbf{x},\mathbf{y}$)

$$= \|\mathbf{u}\|^2 + \cancel{2\mathbf{u}\cdot\mathbf{v}} + \|\mathbf{v}\|^2 + \|\mathbf{u}\|^2 - \cancel{2\mathbf{u}\cdot\mathbf{v}} + \|\mathbf{v}\|^2$$

(because $\mathbf{x}\cdot\mathbf{x} = \|\mathbf{x}\|^2$, and $\mathbf{x}+\mathbf{x} = 2\mathbf{x}$ for all $\mathbf{x}$)

$$= 2\,\|\mathbf{u}\|^2 + 2\,\|\mathbf{v}\|^2$$

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