MATH 54 – HINTS TO HOMEWORK 11

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Here are a couple of hints to Homework 11. Enjoy!

Section 4.5: The dimension of a vector space

4.5.3, 4.5.7, 4.5.11. First express the subspace as the span of some vectors, and then use the following useful trick:

Useful trick: To find a basis of a collection of vectors, form the matrix A whose columns are the vectors, and all you need to do is to find a basis for $Col(A)$. In particular, the dimension of the subspace is the dimension of $Col(A)$ (which is the number of pivots).

4.5.26. Suppose $\mathcal{B} = \{\mathbf{v_1}, \mathbf{v_2}, \cdots \mathbf{v_n}\}$ is a basis for H. What two things can you say about $\mathcal{B}$? Then use the Basis theorem (Theorem 12).

4.5.27. Find an infinite linearly independent set in $\mathbb{P}$. For example, $\{1, x, x^2 \cdots\}$ works!

Section 4.6: The rank of a matrix

Remember that the rank of A is just $dim(Col(A))$. It is also equal to $dim(Row(A))$ and to $Rank(A^T)$ and to the number of pivots of A.

4.6.1, 4.6.5, 4.6.9, 4.6.15. Use the equation $dim(Nul(A)) + Rank(A) = n$. Also, $rank(A)$ is largest when $Nul(A)$ is smallest.

4.6.22. This question is just meant to confuse you with words! All that it says is that if you have an 10×12 matrix, could $Nul(A)$ eveyr be $1-$dimensional? Use rank-nullity to argue that it cannot.

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4.6.33. I urge you to do $4.6.32$ before, it makes this much easier! The point is that if A has rank 1, then all its columns are multiples of the first column. In particular, let $\mathbf{v}$ be the list of the coefficients. For example, if $A = \begin{bmatrix} 1 & -3 & 4 \\ 2 & -6 & 8 \end{bmatrix}$, then let $\mathbf{v} = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, because the second column is -3 times the first one and the third column is 4 times the first one.

If the first column of A is zero, try the second column. If the second column is zero, try the third column. If neither of those hold, then A is the zero matrix, which does not have rank 1.