MATH 54 – HINTS TO HOMEWORK 10

PEYAM TABRIZIAN

Here are a couple of hints to Homework 10. Enjoy!

Section 4.3: Linearly independent sets, bases

Remember that a basis is a linearly independent set which spans the whole space! Equivalently, as set is a basis if the corresponding matrix A is invertible.

4.3.7. It cannot span $\mathbb{R}^3$ because we have 2 vectors in $\mathbb{R}^3$ (and hence it cannot be a basis). To check linear independence, just ask yourself: is the second vector a multiple of the first?

4.3.13. To find $Col(A)$, see where the pivot columns are, and then go back to A and choose precisely those columns (here the first and second column of A)

4.3.21.

(a) **F**
(b) **F**
(c) **T**
(d) **F**
(e) **F**

4.3.32. I love this problem!!! We are given that there exist $c_1, \cdot, c_p$, not all 0, such that:

$$c_1 T(v_1) + \cdot + c_p T(v_p) = \mathbf{0}$$

By linearity of T, this becomes:

$$T(c_1 v_1 + \cdot + c_p v_p) = \mathbf{0} = T(\mathbf{0})$$

But because T is one-to-one, this implies:

$$c_1 v_1 + \cdot + c_p v_p = \mathbf{0}$$

Date: Monday, October 6, 2014.

Since the $c_1, \cdot, c_p$ are not all 0, this shows that the vectors $v_1, \cdot, v_p$ are linearly dependent.

Make sure to understand every step of this (and enjoy how near this is!)

SECTION 4.4: COORDINATE SYSTEMS

Remember: It's easier to figure out $\mathbf{x}$ once we know $[\mathbf{x}]_{\mathcal{B}}$ than the reverse. Also, remember that the change of coordinates matrix is just the matrix whose columns are the elements in $\mathcal{B}$.

It takes a code as its input and tells you which vector corresponds to that code. Its inverse matrix does what you usually want: It produces the coordinates of $\mathbf{x}$

4.4.15.

(a) **T**
(b) **F** (it's $\mathbf{x} = P_{\mathcal{B}} [\mathbf{x}]_{\mathcal{B}}$)
(c) **F** (it's P_2 and $\mathbb{R}^3$ which are isomorphic)

4.4.19. This is also kind of cute. Let $S = \{v_1, \cdots, v_n\}$

<u>Span:</u>

Let $\mathbf{x}$ be an arbitrary vector in V. Then we know that $\mathbf{x}$ has a representation as a linear combination of elements of S, that is, there exist $c_1, \cdots, c_n$ such that:

$$\mathbf{x} = c_1 v_1 + \cdots + c_n v_n$$

And therefore $\mathbf{x}$ is in the span of S, and therefore S spans V (since $\mathbf{x}$ was arbitrary in V).

<u>Linear independence:</u>

Suppose there exist constants $c_1, \cdots, c_n$ such that:

$$c_1 v_1 + \cdots + c_n v_n = \mathbf{0}$$

However, notice that we can write $\mathbf{0} = 0v_1 + \cdots + 0v_n$, and therefore we found *two* ways of writing $\mathbf{0}$ as a linear combinations of elements in S. But since there is only one way of writing $\mathbf{0}$ as a linear combination of elements in S (by assumption), it follows that the c_i *must* all equal to 0. Therefore

$c_1 = \cdots = c_n = 0$, and therefore S is also linearly independent.

4.4.27. Use the basis $\mathcal{B} = \{1, t, t^2, t^3\}$, and compute the coordinates of the 4 polynomials. Then the polynomials are linearly independent if and only if their corresponding vectors are linearly independent!