

MATH 54 – SOLUTION TO 1.2.11

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Row-reduce the matrix (we divided the second row by 3, the third row by 2; and then we subtracted the first row from the second row and the third row):

$$\begin{aligned} \begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix} &\longrightarrow \begin{bmatrix} 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \end{bmatrix} \\ &\longrightarrow \begin{bmatrix} 3 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned}$$

In terms of equations, this becomes $3x_1 - 2x_2 + 4x_3 = 0$, that is $3x_1 = 2x_2 - 4x_3$, hence $x_1 = \frac{2}{3}x_2 - \frac{4}{3}x_3$. Moreover x_2 and x_3 are free, and therefore the general solution to the system is:

$$\begin{cases} x_1 = \frac{2}{3}x_2 - \frac{4}{3}x_3 \\ x_2 \text{ free} \\ x_3 \text{ free} \end{cases}$$

Note: Try to simplify things whenever you can. For example, in the matrix, divide the second row by 3 and the third row by 2, and *then* start your row-reduction process!