

Final Exam – Review – Part II

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1 Systems of differential equations

Problem 1

Solve $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & -3 & 4 \\ 4 & -8 & 7 \end{bmatrix}$$

Note: This is the same matrix as in the second midterm-review session!

Problem 2

Solve $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}$$

2 Theory of differential equations

Problem 3

Find the largest interval (a, b) on which the following differential equation has a unique solution:

$$(x - 2)y'' + \ln(x)y' = \sqrt{3 - x}$$

with $y(1) = 0$, $y'(1) = 2$.

Problem 4

Suppose you happen to know that $f(x) = x \sin(x)e^x$ and $g(x) = \ln(x)e^x$ happen to solve a second-order linear differential equation. Find the general solution of that differential equation.

3 Solving differential equations

Problem 5

Prove that any solution $y(t)$ of $2y''' + 5y'' + 12y' + 5y = 0$ goes to 0 as $t \rightarrow \infty$

Problem 6

Guess the form of a particular solution of:

$$D^3(D - 1)^3 (D^2 + 4)^2 (D^2 - 2D + 5)^2 (y) = f(t)$$

where:

- (a) $f(t) = t^2 e^t$
- (b) $f(t) = t e^t \cos(2t)$
- (c) $f(t) = e^t \cos(3t)$
- (d) $f(t) = e^{2t} \cos(2t)$
- (e) $f(t) = t^2 \sin(2t)$