

MIDTERM 2 – REVIEW – SOLUTIONS

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1. $\dim(W) = 2$

2a. $\text{Rank}(A) = \dim(\text{Col}(A)) = \dim(\text{Row}(A)) = 5$
 $\dim(\text{Nul}(A)) = 1$

2b. Basis for $\text{Row}(A)$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ -13 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

Basis for $\text{Col}(A)$:

$$\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -2 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ -3 \\ 6 \\ 0 \\ -1 \end{bmatrix} \right\}$$

2c. Basis for $\text{Nul}(A)$:

$$\left(\begin{bmatrix} -1 \\ -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right)$$

3. $\begin{bmatrix} 11 \\ 16 \end{bmatrix}$

4a.

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1+2i & 0 \\ 0 & 0 & 1-2i \end{bmatrix}, P = \begin{bmatrix} 1 & -1-i & -1+i \\ 1 & 1+i & 1-i \\ 1 & 2 & 2 \end{bmatrix}$$

4b.

$$C = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & -2 & 1 \end{bmatrix}, P = \begin{bmatrix} 1 & -1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{bmatrix}$$

5. $\mathbf{v} = \begin{bmatrix} 1 \\ -\sqrt{2} \\ 1 \end{bmatrix}$ (or any other nonzero eigenvector corresponding to the eigenvalue $\lambda = -2 + \sqrt{2}$)

6a.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \\ 1 & 0 & 0 \end{bmatrix}$$

6b. $Nul(T) = \{0\}$ (the zero-polynomial), $Ran(T) = Span \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix} \right\}$

7. $\begin{bmatrix} -2 & -2 \\ 0 & -1 \end{bmatrix}$

8a. $\hat{\mathbf{x}} = \begin{bmatrix} 5 \\ \frac{4}{3} \\ \frac{3}{3} \\ \frac{22}{3} \end{bmatrix}$

8b. $\mathbf{x} - \hat{\mathbf{x}} = \begin{bmatrix} -2 \\ \frac{8}{3} \\ 2 \\ -\frac{4}{3} \end{bmatrix}$

8c. $\|\mathbf{x} - \hat{\mathbf{x}}\| = \frac{2}{3}\sqrt{22}$

8d. $\begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

9. (a) **FALSE**
 (b) **FALSE**
 (c) **FALSE**
 (d) **TRUE**
 (e) **FALSE**

- (f) **TRUE**
- (g) **TRUE**
- (h) **TRUE**
- (i) **FALSE**
- (j) **FALSE**
- (k) **FALSE**
- (l) **TRUE**
- (m) **FALSE**
- (n) **TRUE**
- (o) **TRUE**