MIDTERM 1 – REVIEW – PROBLEMS

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1. Linear equations

Problem 1: Solve the following system (or say it has no solutions):

$$\begin{cases} 2x + 2y + z = 2 \\ 3x + 4y + 2z = 3 \\ x + 2y + z = 1 \end{cases}$$

2. Inverses

Problem 2. Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Problem 3. Is the following matrix invertible?

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Hint: This is a one-liner!

3. Linear Transformations

Problem 4. Assume $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points in the plane **clockwise** by $\frac{\pi}{2}$ radians. Find the matrix of T.

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Problem 5. Define $T : P_3 \to P_4$ by:

$$T(p) = \int_0^t p(x)dx$$

(Basically, $T(p)$ is the antiderivative of p without the constant)

(a) Show T is a linear transformation
(b) Find $Nul(T)$. Is T one-to-one?
(c) Find $Ran(T)$. Is T onto P_4?

4. Determinants

Problem 6. Find $\det(A)$, where:

$$A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 0 & 0 & 3 \\ 3 & 1 & 4 & 7 \\ 1 & 1 & 0 & 3 \end{bmatrix}$$

Problem 7. Find $\det(A)$, where:

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \end{bmatrix}$$

5. $Nul(A)$, $Col(A)$

Problem 8.

(a) For the following matrix A, find a basis for $Col(A)$.
(b) Find $Rank(A)$
(c) Find $dim(Nul(A))$
(d) Find a basis for $Nul(A)$

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 9 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ 0 & 0 & 3 & -1 & 1 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

6. Coordinates

Problem 9. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \end{bmatrix} \right\}$. Find $[\mathbf{x}]_{\mathcal{B}}$, where $\mathbf{x} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$

7. Prof. Nadler - Special

Problem 10. For what c are the following systems equivalent?

$$\begin{cases} x - cy = 0 \\ x + z = 0 \end{cases} \quad \text{and} \quad \begin{cases} 2x - y + z = 0 \\ y + z = 0 \end{cases}$$

Problem 11. For what c, d is the following augmented matrix inconsistent:

$$\begin{bmatrix} 3 & -1 & c \\ -9 & 3 & d \end{bmatrix}$$

Problem 12. For what c is $T : \mathbb{R}^2 \longrightarrow \mathbb{R}^3$ injective/surjective?

$$\begin{bmatrix} 1 & 0 \\ 0 & c \\ 0 & 0 \end{bmatrix}$$

Problem 13. For what a is the following matrix invertible? (use determinants)

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & a \\ 4 & 9 & a^2 \end{bmatrix}$$

Problem 14. Find $dim(Col(A))$ and $dim(Nul(A))$, where:

$$A = \begin{bmatrix} 1 & 1 \\ c^2 & 3c - 2 \end{bmatrix}$$

8. True/False Extravaganza

Problem 15.

(a) If Nul(A) $= \{\mathbf{0}\}$, then A is invertible

(b) If $AB = I$, then A is invertible

(c) If A is a 2×3 matrix, then $A\mathbf{x} = \mathbf{0}$ always has a nonzero solution

(d) If $T : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is one-to-one, then T is also onto $\mathbb{R}^n$

(e) If A is $n \times n$ and has n pivots, then the columns of A form a basis for $\mathbb{R}^n$

(f) If W is a subspace of V, and $\mathcal{B}$ is a basis for V, then some subset of $\mathcal{B}$ is a basis for W

(g) The intersection of two subspaces of V is a subspace of V

(h) The union of two subspaces of V is a subspace of V

(i) If A and B are symmetric, then so is $AB + B^T A^T$

(j) A system of 10 equations in 8 unknowns always has a nonzero solution