MIDTERM 1 – REVIEW – SOLUTIONS

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1. $\mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ -\frac{1}{2} \\ 1 \end{bmatrix}$
2. $$A^{-1} = \begin{bmatrix} -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} \\ 0 & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$
3. Not invertible, because the columns of A are linearly dependent
4. $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$
5. Show $T(p+q) = T(p) + T(q)$ and $T(cp) = cT(p)$
   $Nul(T) = \{0\}$, so T is one-to-one, but $Ran(T) = Span\left\{t, t^2, t^3, t^4\right\} \neq P_4$ (calculate $T(a + bt + ct^2 + dt^3)$), so T is not onto P_4
6. -44
7. 0
8. $Rank(A) = 3$, $dim(Nul(A)) = 2$

   Basis for $Col(A)$:

   $$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \\ 3 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ -4 \end{bmatrix} \right\}$$

   Basis for $Nul(A)$:

Date: Thursday, October 2, 2014.

$$\mathcal{B} = \left\{ \begin{bmatrix} \frac{3}{2} \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{9}{2} \\ 0 \\ -\frac{4}{3} \\ -3 \\ 1 \end{bmatrix} \right\}$$

9. $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} -39 \\ 22 \end{bmatrix}$
10. $c = 1$
11. $c \neq -\frac{d}{3}$ (and any d)
12. For injective, need $c \neq 0$. Never surjective.
13. $a \neq 2, 3$
14. If $c = 1, 2$, then $dim(Col(A)) = 1, dim(Nul(A)) = 1$. Else $dim(Col(A)) = 2, dim(Nul(A)) = 0$
15. (a) **FALSE**
    (b) **FALSE**
    (c) **TRUE**
    (d) **TRUE**
    (e) **TRUE**
    (f) **FALSE** (Take $V = \mathbb{R}^2$ with the standard basis, and $W = Span\left\{\begin{bmatrix} 1 \\ 1 \end{bmatrix}\right\}$. You cannot delete vectors from the standard basis to form a basis for W)
    (g) **TRUE**
    (h) **FALSE**
    (i) **TRUE**
    (j) **FALSE**