MATH 54 – MOCK MIDTERM 1

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Name: ______________________________________

Instructions: This is a mock midterm, designed to give you extra practice for the actual midterm. Good luck!!!

1		45
2		15
3		20
4		10
5		10
6		10
7		10
8		20
9		10
Total		150

Date: Thursday, September 25th, 2014.

1. *(45 points, 5 pts each)*

Label the following statements as **T** or **F**.

Make sure to **JUSTIFY YOUR ANSWERS!!!** You may use any facts from the book or from lecture.

(a) If A and B are square matrices, then $(A+B)^{-1} = A^{-1} + B^{-1}$.

(b) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a one-to-one linear transformation, then T is also onto.

(c) If $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ are linearly independent vectors in $\mathbb{R}^n$, then $\{\mathbf{v_1}, \mathbf{v_2}\}$ is linearly independent as well!

(d) If A is an invertible square matrix, then $(A^T)^{-1} = (A^{-1})^T$

(e) If A is a 3×3 matrix with two pivot positions, then the equation $A\mathbf{x} = \mathbf{0}$ has a nontrivial solution.

(f) If A and B are square matrices, then $det(A+B) = det(A) + det(B)$.

(g) If $Nul(A) = \{\mathbf{0}\}$, then A is invertible.

(h) $\mathbb{R}^2$ is a subspace of $\mathbb{R}^3$

(i) If W is a subspace of V and $\mathcal{B}$ is a basis for V, then some subset of $\mathcal{B}$ is a basis for W.

2. *(15 points)* Solve the following system (or say it has no solutions):

$$\begin{cases} x + y + z = 0 \\ 2x + 2z = 0 \\ 3x + y + 3z = 0 \end{cases}$$

3. *(20 points)* Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

4. *(10 points)* What's the next elementary row operation you would use to transform the following matrix in row-echelon form? What is the corresponding elementary matrix?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 2 & 1 \end{bmatrix}$$

5. *(10 points, 5 points each)* Evaluate the following products if they are defined, or say 'undefined'

(a) AB, where:

$$A = \begin{bmatrix} 2 & 5 \\ 0 & 7 \\ -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

(b) AB, where:

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

6. *(10 points)* Suppose $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation which reflects points in the plane about the origin.

(a) (5 points) Find the matrix A of T.

(b) (5 points) Use A to find $T(1, 1)$.

7. *(10 points)* Find the determinant of the following matrix A:

$$A = \begin{bmatrix} 1 & 42 & 536 & 789 & 4201 & 123456789 \\ 0 & 1 & 2012 & 2014 & \pi m & \text{Dolphin} \\ 0 & 0 & 2 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 4 & 0 & 2 & -1 \end{bmatrix}$$

8. *(20 points, 10 points each)*

(a) Find a basis for $Col(A)$, where:

$$A = \begin{bmatrix} 2 & -3 & 6 & 2 & 5 \\ -2 & 3 & -3 & -3 & -4 \\ 4 & -6 & 9 & 5 & 0 \\ -2 & 3 & 3 & -4 & 1 \end{bmatrix}$$

(b) What is $Rank(A)$? What is $Dim(Nul(A))$?

9. *(10 points)* Find a basis for $Nul(A)$ and $Col(A)$, where A is the following matrix:

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$