MATH 54 – MOCK FINAL EXAM

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Name: ______________________________________

Instructions: This is a mock final, designed to give you extra practice for the actual final. Good luck!!!

1		20
2		10
3		10
4		20
5		20
6		20
7		10
8		20
Total		130

Date: Friday, December 9th, 2011.

1. *(20 points)* Use the Gram-Schmidt process to obtain an orthonormal basis of $W = Span\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$, where:

$$\mathbf{v_1} = \begin{bmatrix} 2 \\ 1 \\ 0 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 6 \\ -3 \\ 1 \\ 11 \end{bmatrix}$$

2. *(10 points)* Find a least squares solution to the following system $A\mathbf{x} = \mathbf{b}$, where:

$$A = \begin{bmatrix} 2 & 0 \\ 1 & -1 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

3. *(10 points)* Find the orthogonal projection of t^2 onto the subspace W spanned by $\{1, t\}$, with respect to the following inner product:

$$\langle p, q \rangle = \int_{-1}^{1} p(t)q(t)dt$$

4. *(20 points)*

Find a diagonal matrix D and an orthogonal matrix P such that $A = PDP^T$, where:

$$A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix}$$

5. *(20 points, 2 points each)*

Mark the following statements as **TRUE** or **FALSE**. If the statement is **TRUE**, don't do anything. If the statement is **FALSE**, provide an explicit counterexample.

(a) If A is a 3×3 matrix with eigenvalues $\lambda = 0, 2, 3$, then A must be diagonalizable!

(b) There does not exist a 3×3 matrix A with eigenvalues $\lambda = 1, -1, -1 + i$.

(c) If A is a symmetric matrix, then all its eigenvectors are orthogonal.

(d) If Q is an orthogonal $n \times n$ matrix, then $Row(Q) = Col(Q)$.

(e) The equation $A\mathbf{x} = \mathbf{b}$, where A is a $n \times n$ matrix always has a unique least-squares solution.

(f) If $AB = I$, then $BA = I$.

(g) If A is a square matrix, then $Rank(A) = Rank(A^2)$

(h) If W is a subspace, and $P\mathbf{y}$ is the orthogonal projection of $\mathbf{y}$ onto W, then $P^2\mathbf{y} = P\mathbf{y}$

(i) If $T : V \to W$, where $dim(V) = 3$ and $dim(W) = 2$, then T cannot be one-to-one.

(j) If A is similar to B, then $det(A) = det(B)$.

6. *(20 points)* Solve the following system $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 0 & -1 & 1 \end{bmatrix}$$

7. *(10 points)* Solve the following system $\mathbf{x}' = A\mathbf{x}$, where:

$$A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$$

8. *(20 points)* Solve the following heat equation:

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} & 0 < x < 1, \quad t > 0 \\ u(0,t) = u(1,t) = 0 & t > 0 \\ u(x,0) = x & 0 < x < 1 \end{cases}$$