MATH 54 – MIDTERM 2 STUDY GUIDE

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Note: 1.3.4 means 'Problem 4 in section 1.3'

COMPUTATIONAL QUESTIONS

Chapter 4: Vector Spaces

- Find a basis and state the dimension of a vector space (4.5.7, 4.5.11)
- Find an infinite-dimensional vector space (4.5.27)
- **Given a matrix A, find a basis for $Nul(A)$, $Col(A)$, $Row(A)$, and also find** $Rank(A)$ (4.5.15, 4.6.1, 4.6.3)
- Use the rank-nullity theorem to find $Rank(A)$ etc. (4.6.5, 4.6.9, 4.6.11, 4.6.15, 4.6.22)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ (4.7.5, 4.7.7, 4.7.9)
- Use the change-of-coordinates matrix to find $[\mathbf{x}]_{\mathcal{C}}$ $[\mathbf{x}]_{\mathcal{B}}$ (4.7.1, 4.7.3); also know how to do this for polynomials (4.7.13)

Chapter 5: Diagonalization

- **Find a diagonal matrix D and a matrix P such that $A = PDP^{-1}$, or say A is not diagonalizable** (5.2.9, 5.2.11, 5.3.9, 5.3.11, 5.3.17)
- Show that a given matrix is not diagonalizable (5.3.8, 5.3.11)
- Use the decomposition $A = PDP^{-1}$ to find A^k for any k (5.3.1, 5.3.3)
- **Find the matrix of a linear transformation** (5.4.1, 5.4.3, 5.4.9, 5.4.11, 5.4.17(b)); also know how to do this for more abstract examples, like matrices or polynomials (see for example Peyam's midterm 2)
- Find the complex eigenvalues and eigenvectors of a matrix (5.5.3, 5.5.5)
- Find a matrix C and a matrix P such that $A = PCP^{-1}$ (5.5.13, 5.5.15)
- Interpret what the matrix C means geometrically (5.5.7, 5.5.9, 5.5.11)

Chapter 6: Inner Products and Norms

NOTE: Although section 6.7 (inner product spaces) is technically not on the exam, it's just an application of the techniques in this chapter to more abstract vector spaces, so you **ARE** supposed to know it!

- Determine if a set is orthogonal, or orthonormal (6.2.3, 6.2.19, 6.2.21)

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- Find all vectors which are perpendicular to a certain set of vectors (see Question 2 on Practice Quiz 9)
- Find a basis for $W^{\perp}$ (all you do is find a basis for W, and find all vectors which are perpendicular to the vectors in the basis, just like the question above)
- Find the orthogonal projection of $\mathbf{x}$ on a subspace W. Use this to write $\mathbf{x}$ as a sum of two orthogonal vectors, and to find the smallest distance between $\mathbf{x}$ and W (6.2.11, 6.3.3, 6.3.5, 6.3.1, 6.3.7, 6.2.15, 6.3.11)
- Given an orthogonal basis $\mathcal{B}$, find $[\mathbf{x}]_{\mathcal{B}}$ (6.2.7, 6.2.9)

TRUE/FALSE EXTRAVAGANZA

Do the following set of T/F questions (you can find solutions to the starred problems in the hints to HW 11-17):

4.5.19, 4.5.20, 4.6.17, 4.6.18, 4.7.11*, 4.7.12, 5.1.21*, 5.1.22, 5.2.21*, 5.3.21*, 5.3.22, 6.1.19*, 6.1.20, 6.2.23*, 6.2.24, 6.3.21*, 6.3.22

CONCEPTS

Understand the following concepts:

- Basis, Dimension (4.5)
- $Nul(A)$, $Col(A)$, $Row(A)$, Rank (4.6)
- Rank-Nullity Theorem (4.6)
- Change of coordinates matrix (4.7)
- Eigenvalues, Eigenvectors, Characteristic polynomial (5.1 - 5.3)
- A is similar to B (5.2)
- Diagonalizable, Diagonalization Theorem (Theorem 5 in section 5.3)
- Matrix of a Linear transformation (5.4)
- Inner products, Norms, Orthogonal vectors, Orthogonal Matrix (6.1)
- $W^{\perp}$ (6.1)
- $(Row(A))^{\perp} = Nul(A), (Col(A))^{\perp} = Nul(A^T)$ (Theorem 3 in 6.1)
- Orthogonal projection (6.2, 6.3)