1. Decide if the following statements are ALWAYS TRUE or SOMETIMES FALSE.

1) The matrices $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ are similar.

TRUE because.. $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ gives

$$A = PBP^{-1}.$$

2) The matrices $A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$ are similar.

FALSE because.. they have different characteristic equations.

3) For 2×2 matrices A and B, if $\mathbf{v}$ is an eigenvector of AB, then $B\mathbf{v}$ is an eigenvector of A.

FALSE because.. we have a counterexample. For example, A can be any matrix and $B = \mathbf{0}$, the zero matrix, can be a counterexample. Note that, in such a case, $AB = \mathbf{0}$, so $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is an eigenvector. However, $B\mathbf{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ is not an eigenvector of A since we do not let the zero vector be an eigenvector in any cases.

4) If a 3×3 matrix A is diagonalizable with eigenvalues ± 1, then it is an orthogonal matrix.

FALSE because.. $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{bmatrix}$ is diagonalizable. However, it is not an orthogonal matrix because its columns do not form an orthonormal basis for $\mathbb{R}^3$.

5) If $||\mathbf{u} - \mathbf{v}|| = ||\mathbf{u} + \mathbf{v}||$, then $\mathbf{u}$ and $\mathbf{v}$ are orthogonal.

TRUE because.. see the textbook Chapter 6.1 - Orthogonal Vectors.