MATH 54 – MIDTERM 1 STUDY GUIDE

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Note: 1.3.4 means 'Problem 4 in section 1.3'

1. Computational questions

- Solve a system of equations, or determine if there are no solutions. Try to write your solutions in vector form. (1.1.11, 1.1.13, 1.2.11, 1.2.13, 1.4.11, 1.5.5).
- Determine if a given vector $\mathbf{b}$ is a linear combination of other vectors (1.3.11, 1.3.15)
- Determine whether a set of vectors is linearly dependent or independent (1.7.5, 1.7.7, 1.7.11, 1.7.15, 1.7.17)
- Given a linear transformation T and a vector $\mathbf{b}$, determine whether $\mathbf{b}$ is in the image of T (1.8.3, 1.8.9)
- Find the matrix of a given linear transformation T (1.9.1, 1.9.3, 1.9.5, 1.9.9, 1.9.11, 1.9.17)
- Determine if a linear transformation is one-to-one or onto (1.9.25)
- Given A and B, find AB, or say 'it does not exist' (2.1.5, 2.1.6, 2.1.9)
- Find the inverse of a matrix A (2.2.1, 2.2.3, 2.2.31, 2.3.3, 2.3.7)
- Given A, find a basis for $Nul(A)$ and find a basis for $Col(A)$, and state their dimensions (2.6.23, 2.6.26, 2.7.9, 2.7.12)
- Do problems involving the rank-nullity theorem (2.7.16, 2.7.21)
- Calculate the determinant of a matrix, possibly using row-reductions (3.1.9, 3.1.11, 3.1.13, 3.2.5, 3.2.7, 3.2.11, 3.2.21)
- Solve questions using the fact that $det(AB) = det(A)det(B)$ (3.2.31, 3.2.33, 3.2.34, 3.2.35)
- Solve a system using Cramer's rule (3.3.1, 3.3.3, 3.3.5)
- Find inverses using determinants (3.2.13)
- Calculate volumes using determinants (3.3.21, 3.1.31, 3.3.32)
- Determine if a set is a vector space or not (4.1.1, 4.1.3, 4.1.9, 4.1.17, 4.2.7, 4.2.9, 4.2.11)
- Determine whether a set is linear independent or dependent (4.3.3, 4.4.27)
- Given $[\mathbf{x}]_{\mathcal{B}}$, find $\mathbf{x}$, and vice-versa (4.4.1, 4.4.3, 4.4.5, 4.4.7, 4.4.11)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis in $\mathbb{R}^n$ (4.4.9)

Note: Remember to review Quizzes 1-6 as well!

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Note: I noticed that Prof. Nadler really seems to like questions with constants involved, i.e. find c such that the following set is linearly independent, as in 1.1.20, 1.3.15, or 1.7.11. Make sure to review the problems with constants he covered in his lecture notes. Examples include:

- For what c are two systems equivalent (Lecture 1)
- For what c is a system inconsistent (Lecture 1)
- For what c is a vector $\mathbf{u}$ a linear combo of two (or three) other vectors (Lecture 2)
- For what $\mathbf{b}$ is $A\mathbf{x} = \mathbf{b}$ consistent (Lecutre 3)
- For what c is a linear transformation injective/surjective (Lecture 5)
- For what c is a matrix invertible (Lecture 5 and Lecture 9)
- Find dim(Col(A)) and dim(Nul(A)) where A has constants c in them, (Lecture 7 and 9)

2. True/False Extravaganza

Do the following set of T/F questions: 1.4.23, 1.5.24*, 1.7.21*, 1.7.22, 1.8.21*, 1.8.22, 1.9.23, 1.9.24*, 2.2.9, 2.3.11, 2.3.12, 2.6.21*, 2.6.22, 2.7.17, 3.2.27*, 4.1.23, 4.1.24*, 4.2.25*, 4.2.26, 4.3.21*, 4.3.22, 4.4.15* (The ones with * next to them have solutions in the Homework-Hints)

3. Concepts

Understand the following concepts:

- Pivots (1.2.23, 1.2.26, 1.5.29)
- Span (1.3.22, 1.3.25, 1.4.17, 1.4.29)
- Linear independence (1.7.33, 1.7.35, 1.7.36)
- One-to-one and onto (1.9.25, Quiz 2)
- Invertible matrices (2.2.13, 2.2.21)
- Implications of invertibility (2.3.11, 2.3.15, 2.3.19, 2.3.30).
  Know all the implications of the Invertible Matrix Theorem (including page 144)
- Subspace, Basis (2.6.1, 2.6.3, 2.6.5, 2.6.7, 2.6.17, 2.6.19)
- $Nul(A)$, $Col(A)$, **The Rank Theorem** (2.6.15, 2.6.16, 2.6.21, 2.6.23, 2.6.24, Problem 1 on Quiz 5)
- Vector space, Subspace (4.1.1, 4.1.3, Table on page 192, Problem 1 on Quiz 6)
- Basis, Dimension (4.3.3, 4.3.15, 4.3.23, 4.3.24, 4.3.29, know the Basis Theorem on page 144)
- Coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$ (4.4.3, 4.4.5, 4.4.14)