MATH 54 – FINAL EXAM STUDY GUIDE

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This is the study guide for the final exam! It says what it does: to guide you with your studying for the exam! The terms in **boldface** are more important than others, so make sure to study them in detail!

Note: Remember that the final exam is *cumulative*. It officially covers up to and including 10.3, but we actually could also ask you questions from sections $10.4 - 10.6$. You don't have to read 10.5 and 10.6, just know how to combine the methods in 10.2 and 10.4.

Suggestion: Start with the differential equations-part, since it's fresh in your mind. Once you're done with that, turn your attention to the linear algebra-part.

DIFFERENTIAL EQUATIONS

Chapter 4: Linear Second-order equations

- **Find the general solution to a second-order differential equation, possibly including complex roots, repeated roots, or initial conditions** (4.2.1, 4.2.5, 4.2.15, 4.3.5, 4.3.11, 4.3.23)
- Determine if two functions are linearly independent or linearly dependent (4.2.27, 4.2.30)
- Solve equations using undetermined coefficients (4.4.14, 4.4.27, 4.4.31, 4.5.27, 4.5.35)
- You do **NOT** need to know variation of parameters

Chapter 6: Theory of higher-order linear differential equations

- **Find the largest interval on which a differential equation has a unique solution** (6.1.3, 6.1.5)

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- **Determine if a set of functions is linearly independent or linearly dependent** (6.1.9, 6.1.14, 6.1.13, 6.2.25)
- Show that given functions form a fundamental solution set for a differential equation (6.1.15, 6.1.17, 6.1.19)
- **Find the general solution of a higher-order differential equation, possibly including initial conditions** (6.2.7, 6.2.11, 6.2.15, 6.2.17, 6.2.19, 6.2.20); know the technique using the rational roots theorem and long-division.

Chapter 9: Matrix methods for linear systems

- Convert a higher-order equation into a system of differential equations (9.1.13)
- **Determine if a set of vector functions is linearly independent or linearly dependent**, using the Wronskian (9.4.16, 9.4.17)
- Determine if vector functions form a fundamental solution set for $\mathbf{x}' = A\mathbf{x}$ (see Practice Quiz 13)
- **Find the general solution to $\mathbf{x}' = A\mathbf{x}$, possibly including complex eigenvalues** (9.5.13, 9.5.15, 9.5.21, 9.5.31, 9.5.33, 9.6.1, 9.6.9, 9.6.13)
- Solve $\mathbf{x}' = A\mathbf{x}$, where A is not diagonalizable (9.5.35, 9.5.36); we will **ONLY** ask you about the 2×2-case.
- Do **NOT** need to know about: Sketching trajectories (as in 9.5.17), Normal frequencies (as in 9.6.19)

Chapter 10: Partial differential equations

- **Calculate the Fourier series of a function f on a given interval, ~~and determine to which function that Fourier series converges~~** (10.3.9, 10.3.11, 10.3.15, 10.3.17, 10.3.19, 10.3.23)
- Calculate the Fourier cosine/sine series for a function f, ~~and determine to which function that Fourier series converges to~~ (10.4.5, 10.4.7, 10.4.13). You actually need to know this because it was covered in lecture. It's actually the same thing as 10.3, except that you multiply everything by 2, and you only integrate from 0 to L.
- **Using separation of variables, solve the heat and wave equation, subject to various boundary/initial conditions** (10.2.15, 10.2.21, 10.5.1, 10.5.3, 10.5.5, 10.5.7, 10.6.1, 10.6.3). Again, this is fair game for the exam, because it's just a combination of 10.2 and 10.3

(and what was covered in lecture), but you don't need to read 10.5 and 10.6 to do this.

LINEAR ALGEBRA

Suggestion: Ignore chapters $1, 2, 3$, because you basically already know how to do them! Instead, focus on chapters $4, 5, 6, 7$

Chapter 4: Vector Spaces

- Determine if a set is a vector space or not (4.1.3, 4.1.9, 4.2.7, 4.2.9)
- Determine whether a set is linear independent or dependent (4.3.3, 4.4.27)
- Find a basis and state the dimension of a vector space (4.5.7, 4.5.11)
- **Given a matrix A, find a basis for $Nul(A)$, $Col(A)$, $Row(A)$, and also find $Rank(A)$** (4.5.15, 4.6.1, 4.6.3)
- Use the rank-nullity theorem to find $Rank(A)$ etc. (4.6.5, 4.6.9, 4.6.11, 4.6.15, 4.6.22)
- Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ (4.4.1, 4.4.3, 4.4.5, 4.4.7, 4.4.11, 4.7.5, 4.7.7, 4.7.9)
- Use the change-of-coordinates matrix to find $[\mathbf{x}]_{\mathcal{C}}$ $[\mathbf{x}]_{\mathcal{B}}$ (4.7.1, 4.7.3); also know how to do this for polynomials (4.7.13)

Chapter 5: Diagonalization

Don't focus *too* much on this chapter, because it's almost the same as Chapter 9.

- **Find a diagonal matrix D and a matrix P such that $A = PDP^{-1}$, or say A is not diagonalizable** (5.2.9, 5.2.11, 5.3.9, 5.3.11, 5.3.17)
- Show that a given matrix is not diagonalizable (5.3.8, 5.3.11)
- **Find the matrix of a linear transformation** (5.4.1, 5.4.3, 5.4.9, 5.4.11, 5.4.17(b)); also know how to do this for more abstract examples, like matrices or polynomials (see for example Peyam's midterm 2)

Chapter 6: Inner Products and Norms

- Find a basis for $W^{\perp}$ (all you do is find a basis for W, and find all vectors which are perpendicular to the vectors in the basis, just like the question above)
- Find the orthogonal projection of $\mathbf{x}$ on a subspace W. Use this to write $\mathbf{x}$ as a sum of two orthogonal vectors, and to find the smallest

distance between $\mathbf{x}$ and W (6.2.11, 6.3.3, 6.3.5, 6.3.1, 6.3.7, 6.2.15, 6.3.11)

- **Use the Gram-Schmidt process to produce an orthonormal basis of a subspace W spanned by some vectors** (6.4.9, 6.4.11)
- Find the QR-factorization of a matrix (6.5.13, 6.5.15). This is also fair game for the exam, because it was covered in lecture
- **Find the least-squares solution (and least-squares error) of an inconsistent system of equations** (6.5.1, 6.5.3, 6.5.7, 6.5.9, 6.5.11)
- Find inner products, lengths, and orthogonal projections of functions f and g using fancier inner products $\langle f, g\rangle$ (6.7.3, 6.7.5, 6.7.7, 6.7.9, 6.7.11, 6.7.21, 6.7.23)
- Use the Gram-Schmidt process to find an orthonormal basis of **functions** (6.7.25, 6.7.26)
- Remember the Cauchy-Schwarz inequality (6.7.19, 6.7.20)

Chapter 7: Symmetric matrices and quadratic forms

- **Given a symmetric matrix A, find an diagonal matrix D and an orthogonal matrix P such that $A = PDP^T$** (7.1.13, 7.1.15, 7.1.17)

TRUE/FALSE EXTRAVAGANZA

Do the following set of T/F questions: 1.4.23, 1.5.24*, 1.7.21*, 1.7.22, 1.8.21*, 1.8.22, 1.9.23, 1.9.24*, 2.2.9, 2.3.11, 2.3.12, 2.6.21*, 2.6.22, 2.7.17, 3.2.27*, 4.1.23, 4.1.24*, 4.2.25*, 4.2.26, 4.3.21*, 4.3.22, 4.4.15*, 4.5.19, 4.5.20, 4.6.17, 4.6.18, 4.7.11*, 4.7.12, 5.1.21*, 5.1.22, 5.2.21*, 5.3.21*, 5.3.22, 6.1.19*, 6.1.20, 6.2.23*, 6.2.24, 6.3.21*, 6.3.22, 6.4.17*, 6.4.18, 6.5.17*, 6.5.18, 7.1.25*, 7.1.26 (The ones with * next to them have solutions in the Homework-Hints)

CONCEPTS

Understand the following concepts:

- Pivots (1.2, 1.5)
- Span (1.3, 1.4)
- Linear independence (1.7)
- One-to-one and onto (1.9)
- Invertible matrices (2.2)
- Implications of invertibility (2.3).
  Know all the implications of the Invertible Matrix Theorem (including page 144)
- Subspace, Basis (2.6)
- $Nul(A)$, $Col(A)$, **The Rank Theorem** (2.6, 4.6)
- Vector space, Subspace (4.1)
- Basis, Dimension (4.3, 4.5)
- Coordinates of $\mathbf{x}$ with respect to $\mathcal{B}$ (4.4)
- Basis, Dimension (4.5)
- Change of coordinates matrix (4.7)
- Eigenvalues, Eigenvectors, Characteristic polynomial (5.1 - 5.3)
- A is similar to B (5.2)
- Diagonalizable, Diagonalization Theorem (Theorem 5 in section 5.3)
- Matrix of a Linear transformation (5.4)
- Inner products, Norms, Orthogonal vectors, Orthogonal Matrix (6.1)
- $W^{\perp}$ (6.1)
- $(Row(A))^{\perp} = Nul(A), (Col(A))^{\perp} = Nul(A^T)$ (Theorem 3 in 6.1)
- Orthogonal projection (6.2, 6.3)
- Least-squares solution (6.4)
- Inner product, inner product space (6.7)
- Cauchy-Schwarz and Triangle Inequality (6.7)
- Orthogonally diagonalizable (7.1)
- Existence and Uniqueness Theorem for differential equations (6.1)
- Linear independence of functions/vector functions and Wronskian/fundamental matrix (4.2, 6.1, 9.4)
- Fundamental solution set (6.1, 9.4)
- Separation of variables (10.2)
- Fourier series, Fourier cosine/sine series (10.3, 10.4)
- Pointwise convergence of Fourier series (10.3)
- Orthogonal expansions (10.3)