(1) **Note:** The (2, 5)th entry of \( A \) should be a \(-4\), not a 4 (the minus-sign is faint!)

Row-reduce \( A \) until you get:

\[
\begin{bmatrix}
3 & 18 & 10 & 2 & 7 \\
0 & 0 & 4 & 11 & 19 \\
0 & 0 & 0 & 3 & -13 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

(a) Basis for \( \text{Row}(A) \):

\[
B = \left\{ \begin{bmatrix} 3 \\ 18 \\ 10 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \\ 11 \\ 19 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 3 \\ -13 \end{bmatrix} \right\}
\]

(b) Basis for \( \text{Col}(A) \):

\[
B = \left\{ \begin{bmatrix} 3 \\ 1 \\ 2 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} 10 \\ 2 \\ 8 \\ 0 \\ 12 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 5 \\ 10 \\ -21 \end{bmatrix} \right\}
\]

(2) \[ [x]_B = \begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix} \]

Think of this as a change-of-basis problem!

Let \( P = [b_1 \ b_2 \ b_3] = [[b_1]_E \ [b_2]_E \ [b_3]_E] \)

Then: \( P = E \iff B. \)

Hence:

\[
[x]_E = E \iff B [x]_B = P [x]_B
\]

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So:

\[ [x]_B = P^{-1}x \]

(3) \( y = 2x - \frac{1}{2} \)

In other words, try to solve \( Ax = b \) in the least squares sense, where:

\[
A = \begin{pmatrix}
1 & 1 \\
2 & 1 \\
3 & 1 \\
4 & 1
\end{pmatrix},
\]

\[
b = \begin{pmatrix}
2 \\
3 \\
5 \\
8
\end{pmatrix}
\]

(4) \( B = \left\{ \begin{pmatrix}
1 \\
2 \\
3 \\
4
\end{pmatrix}, \begin{pmatrix}
1 \\
2 \\
3 \\
1
\end{pmatrix} \right\} \)

**CAREFUL!** You should get \( w_3 = 0 \), but do **NOT** include it in your basis!

What this says is that the first two vectors are linearly independent, but the third one isn’t!

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(6) (a) \( B = \begin{pmatrix}
1 \\
0 \\
0 \\
-3
\end{pmatrix}, \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix} \)

(b) 5

(7) This is **HARD!!!**

Suppose \( \mathbf{v} \) is an eigenvector of \( B \) with eigenvalue \( \lambda \). Then \( B\mathbf{v} = \lambda \mathbf{v} \).

But then:

\[
A\mathbf{v} = B^2\mathbf{v} = B(\lambda \mathbf{v}) = \lambda B\mathbf{v} = \lambda \lambda \mathbf{v} = \lambda^2 \mathbf{v}
\]

So if \( \mathbf{v} \) is an eigenvector of \( A \), then \( \mathbf{v} \) is also an eigenvector of \( B \), but with eigenvalue \( \lambda^2 \).

But if you do the calculations, you find that \( A \) has eigenvalues 1 and 4 with corresponding eigenvectors:
\[
\begin{bmatrix}
1 \\
1
\end{bmatrix},
\begin{bmatrix}
2 \\
1
\end{bmatrix}
\]

Hence, by what we said before, \(B\) has the same eigenvectors, but with eigenvalues \(\sqrt{1} = 1\) and \(\sqrt{4} = 2\).

This means that:

\[B = PDP^{-1} = \begin{bmatrix}
3 & -2 \\
1 & 0
\end{bmatrix}\]

Where \(P = \begin{bmatrix}
1 & 2 \\
1 & 1
\end{bmatrix}\), \(D = \begin{bmatrix}
1 & 0 \\
0 & 2
\end{bmatrix}\)

**Note:** The point is: If \(A = PDP^{-1}\), then \(A^k = PD^kP^{-1}\), and this holds for ANY \(k\), even fractions! In this problem, you found \(B = \sqrt{A} = A^{\frac{1}{2}} = PD^{\frac{1}{2}}P^{-1}\).