Mathematics 54. **Spring Semester 2008**

Practice Midterm 2

Problem 1. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Calculate its characteristic polynomial. Determine bases for each of the eigenspaces of A. Diagonalize A if possible.

Problem 2. Find numbers x and y so that the product $\begin{bmatrix} 1 & 2 \\ 2 & 4 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ is as close as possible to the vector $\begin{bmatrix} 20 \\ 40 \\ 80 \end{bmatrix}$.

Problem 3. Let $W \subset R^4$ be the subspace

$$W = span\{u_1, u_2, u_3\}, \qquad u_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 0 \end{bmatrix} u_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 1 \end{bmatrix} u_3 = \begin{bmatrix} 0 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$

Find an orthogonal basis for W.

Problem 4. Give an example of a $3x3$ matrix with eigenvalues $1, 2 \pm 3i$.

Problem 5.TRUE or FALSE (justify your answers)

a) If A is an $n \times n$ matrix so that $A^2 = A$ then its only possible eigenvalues are 0 and 1.

b) Let A be an $n \times n$ matrix. If there is an orthonormal basis in R^n consisting of eigenvectors of A then A must be symmetric.

c) Any triangular matrix is diagonalizable.

d) If A is a square matrix with $A^T A = I$ then its rows are orthogonal.