MIDTERM 2 (RIBET) - ANSWER KEY

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(1) Notice that the matrix is already in row-echelon form!

(a) Basis for Row(A):

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 8 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ -1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ -3 \\ -10 \end{bmatrix} \right\}$$

(b) Basis for Col(A):

$$\mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 8 \\ 6 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ -1 \\ 0 \end{bmatrix} \right\}$$

(c) Basis for Nul(A):

$$\mathcal{B} = \left\{ \begin{bmatrix} 8 \\ -5 \\ -18 \\ 6 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -7 \\ 60 \\ 0 \\ 6 \end{bmatrix} \right\}$$

Note: There are many ways to do this, I just row-reduced A further until I got the reduced row-echelon form of A, then I solved for $\mathbf{x}$, and I multiplied the resulting vectors by 6 in order to get rid of the fractions!

(2) The eigenvalues are $\lambda = 4, 2$

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$$

YES (A is 3×3 and you just found 3 linearly independent eigenvectors).

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(3) 2, Apply the Gram-Schmidt process to $\{\mathbf{v_1}, \mathbf{v_2}\}$ to get:

$$\mathbf{w_1} = \begin{bmatrix} 1 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \mathbf{w_2} = \begin{bmatrix} 0 \\ 10 \\ -2 \\ -8 \end{bmatrix}$$

And $\mathcal{B} = \{\mathbf{w_1}, \mathbf{w_2}\}$ is an orthogonal basis for W.

(4) -125

(5) Suppose $\mathbf{y}$ is in $Col(A)$.

By definition of $Col(A)$, this means that there is an $\mathbf{x}$ such that $A\mathbf{x} = \mathbf{y}$.

But then:

$$A\mathbf{y} = A\left(A\mathbf{x}\right) = A^2\mathbf{x} = A\mathbf{x} = y$$

(where in the middle step, we used the fact that $A^2 = A$)

If $Nul(A) = \{\mathbf{0}\}$, then A is invertible (since A is square).

Then multiply both sides of $A^2 = A$ by A^{-1} to get $A = I$.