MIDTERM 2 (BERGMAN) - ANSWER KEY

PEYAM RYAN TABRIZIAN

(1) (a) $\lambda = -5, 0, 1$

(b)

$$\mathcal{B} = \left\{ \begin{bmatrix} -5 \\ -4 \\ 8 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$$

Note: Any multiple of those vectors works as well!

(c)

$$P = \begin{bmatrix} -5 & 0 & 1 \\ -4 & 2 & 2 \\ 8 & 1 & 2 \end{bmatrix}$$

Note: The order doesn't matter, as long as you match the correct eigenvector with the corresponding eigenvalue!

(d)

$$D = \begin{bmatrix} -5 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Note: The formula you usually know is $A = PDP^{-1}$, but this implies $D = P^{-1}AP$.

(2) (a) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ (all that this says is find a nonzero matrix with nonzero nullspace)

(c) Any positive multiple of the standard inner product works!

$$\left\langle \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \right\rangle = 2(x_1y_1 + x_2y_2)$$

Date: Friday, October 21st, 2011.

(d) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. If you want a fancier example, try $A = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$.

CAREFUL: The columns of A have to be ortho**NORMAL**.

(e) $V = \mathbb{R}^2$, for S just choose a set which has 'too' many elements (at least two of them being linearly independent), such as:

$$S = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right\}$$

(f)

$$\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\sqrt{5}} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix}$$

(3) This problem is a bit silly if you think about it!

By definition of 'consistent', the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution $\mathbf{x_b}$. Similarly the equation $A\mathbf{x} = \mathbf{c}$ also has a solution $\mathbf{x_c}$.

In order for $A\mathbf{x} = \mathbf{b} + \mathbf{c}$ to be consistent, it has to have at least one solution. What could that solution be? Try $\tilde{\mathbf{x}} = \mathbf{x_b} + \mathbf{x_c}$. Then:

$$A\tilde{\mathbf{x}} = A(\mathbf{x_b} + \mathbf{x_c}) = A\mathbf{x_b} + A\mathbf{x_c} = \mathbf{b} + \mathbf{c}$$

Hence $\tilde{\mathbf{x}}$ solves $A\mathbf{x} = \mathbf{b} + \mathbf{c}$, hence this equation has at least one solution, hence it is consistent!

(4) $\hat{\mathbf{x}} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$

(5) Ignore!