FINAL EXAM (TRANG) - ANSWER KEY

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Multiple Choice:

(1) B (f_1, f_2, f_3 have to be solutions to a differential equation)
(2) A
(3) A
(4) C
(5) C
(6) A
(7) B
(8) A
(9) C (I think there's a typo in this question, the 1 should be an x)
(10) A
(11) A
(12) B
(13)
  (13.1) C (namely, $Proj(\mathbf{b})$ on $Col(A)$, but notice that any vector in $Col(A)$ has its second entry equal to 0)
  (13.2) B
(14) A
(15) A
(16)
  (16.1) B
  (16.2) B
(17) B(again, the 2 should be a $2x$)
(18) A
(19) A
(20) A
(21) A
(22) B (I got $\begin{bmatrix} 0 \\ e^t + 4te^t \\ e^t + 2te^t \end{bmatrix}$)
(23) B
(24) A
(25) A

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(26) Both methods should give you the same answer, namely:

$$y(t) = \frac{7}{3}e^{t} - \frac{10}{3}e^{2t} - (t-1)e^{2t} + \frac{t^2}{2}e^{2t}$$

(27)

$$\mathbf{x}(t) = \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$$

(use generalized eigenvectors, then undetermined coefficients. Then, if you plug in $t = 0$ in your general solution, you should get that $A = B = 0$)

(28)

$$[T]_{\mathcal{C}\leftarrow\mathcal{B}} = \begin{bmatrix} 0 & -1 & -2 \\ 0 & 1 & 3 \\ 1 & 1 & 0 \end{bmatrix}, \qquad \left[T(1+t+t^2)\right]_{\mathcal{C}} = \begin{bmatrix} -3 \\ 4 \\ 0 \end{bmatrix}$$

(29)

$$\hat{\mathbf{v}} = \begin{bmatrix} \frac{1}{3} \\ -\frac{5}{3} \\ \frac{4}{3} \end{bmatrix}, \qquad d = \frac{\sqrt{42}}{3}$$

Hint: The plane is spanned by $\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$. Make sure to apply Gram-Schmidt to those vectors first before applying the projection-formula!