Optimization

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How to solve optimization problems

1) **Draw a picture!**, labeling all your variables. This time, you **can** put numbers on your picture (in contrast to related rates problems)

2) Find a function of **one variable** which you want to maximize/minimize (here is where you use *all* the info that is given to you)

3) Find the constraint. Sometimes, there's an 'open interval' constraint like $x > 0$, and sometimes there's a 'closed interval' constraint, like $3 \leq x \leq 6$)

4) Find the absolute maximum or minimum of your function in 2) given the constraint in 3). If your constraint is a closed interval, use the closed interval method from section 4.1. In all the other cases, just solve for $f'(x) = 0$ and write 'by FDTAEV'.

Note: Sometimes, it's useful to maximize the *square* of your function instead of your function (in order to avoid square roots)

List of tricks (unlike for related rates, solving an optimization problem relies more on ingenuity than on memorizing formulas)

- Formula for the distance between two points (x, y) and (x_0, y_0):

$$D = \sqrt{(x - x_0)^2 + (y - y_0)^2}$$

- Formulas for areas and/or volumes
    - Volume of a cone: $V = \frac{\pi}{3} r^2 h$
    - Volume of a cylinder: $V = \pi r^2 h$
    - Surface area of a cylinder (w/o the top): $S = 2\pi r h$

Problem 1

Find two positive numbers whose sum is 100 and whose product is a minimum.

Problem 2

Find the dimensions of a rectangle with area 100 m^2 and whose perimeter is as small as possible. What kind of a rectangle is this?

Problem 3

[4.7.13] If 1200 m^2 of material is available to make a box with a square base and an open top, find the largest possible volume of the box.

Problem 4

[4.7.11] A farmer wants to fence an area of 600 square feet in a rectangular field and then divide it in half with a fence parallel to one of the sides of the rectangle. Find the dimensions of the fence that minimize the cost of building the fence?

Problem 5

Find the point on the line $2x + y = 1$ that is closest to the point $(-3, 1)$

Problem 6

[4.7.21] Find the dimensions of the rectangle of largest area that can be inscribed in a circle of radius r. What kind of a rectangle is this?

Problem 7

[4.7.24] Find the dimensions of the rectangle of largest area that has its base on the $x-$ axis and its other two vertices above the $x-$ axis and lying on the parabola $y = 8 - x^2$.

Problem 8

[4.7.35] A cylindrical can without a top is made to contain 1000 cm^3 of liquid. Find the dimensions that will minimize the cost of the metal to make the can.