Volume - Extravaganza

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1 Disk Method

1.1 Rotating about the $x-$axis, or $y = k$ (typical case)

$$V = \int_a^b \pi \left(f(x) - k\right)^2 dx$$

1.2 Rotating about the $y-$axis, or $x = k$ (weird case, need to solve for x in terms of y)

$$V = \int_a^b \pi \left(f(y) - k\right)^2 dy$$

Tip: Use this when your region is attached to your axis of rotation

2 Washer Method

2.1 Rotating about the $x-$axis, or $y = k$ (typical case, vertical washers)

$$V = \int_a^b \pi \left((\text{Outer})^2 - (\text{Inner})^2\right) dx$$

Where Outer = Bigger function $-$ k, Inner = Smaller function $-$ k

2.2 Rotating about the $y-$axis, or $x = k$ (weird case, horizontal washers, need to solve for x in terms of y)

$$V = \int_a^b \pi \left((\text{Outer})^2 - (\text{Inner})^2\right) dy$$

Where Outer = Rightmost function $-$ k, Inner = Leftmost function $-$ k

Tip: Use this when the disk method fails, i.e. your region is **not** glued to your axis of rotation.

Note: Make sure your answer is **positive**. In some rare cases (see section), what *you* think is the bigger function is *actually* the smaller function! Basically, if you get a negative answer, the correct answer is minus your answer!

3 Shell method

3.1 Rotating about the $y-$axis, or $x = k$ (typical case, vertical rectangles/shells)

$$V = \int_a^b \mathbf{2}\pi \left|x - k\right| (\text{Bigger - Smaller}) dx$$

3.2 Rotating about the $x-$axis, or $y = k$ (weird case, horizontal rectangles/shells, need to solve for x in terms of y)

$$V = \int_a^b \mathbf{2}\pi \left|y - k\right| (\text{Rightmost - Leftmost}) dy$$

Tip: Use it when the washer method fails/is too complicated, typically when you can't solve for x in terms of y. It's also very helpful for more abstract problems!

Tip: Here's an easy way to memorize those formulas: If you're rotating about $x = k$, then $x - k = 0$, so your formula should involve |**x - k**|. Similarly, for $y = k$, $y - k = 0$, so your formula should involve |**y - k**|.

4 Other method

If you're given that the cross-sections are triangles or squares or other familiar geometric objects (see practice final), you need to use the original definition of volume:

4.1 Vertical Slices

$$V = \int_a^b A(x) dx$$

4.2 Horizontal Slices

$$V = \int_a^b A(y) dy$$