Optimization

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How to solve optimization problems

1) **Draw a picture!**, labeling all your variables. This time, you **can** put numbers on your picture (in contrast to related rates problems)

2) Find a function of **one variable** which you want to maximize/minimize (here is where you use *all* the info that is given to you)

3) Find the constraint. Sometimes, there's an 'open interval' constraint like $x > 0$, and sometimes there's a 'closed interval' constraint, like $3 \leq x \leq 6$)

4) Find the absolute maximum or minimum of your function in 2) given the constraint in 3). If your constraint is a closed interval, use the maximization technique from section 4.1. In all the other cases, use the following theorem:

Theorem (First Derivative Test for Absolute Extreme Values)**.** *Suppose c is a critical number of f. Then:*

- *If $f'(x) > 0$ for all $x < c$ and $f'(x) < 0$ for all $x > c$, then $f(c)$ is the absolute maximum value of f*

- *If $f'(x) < 0$ for all $x < c$ and $f'(x) > 0$ for all $x > c$, then $f(c)$ is the absolute minimum value of f*

List of tricks

- **Formulas for areas and/or volumes**
- Formula for the distance between two points
- Definition of $\sin(\theta)$, $\cos(\theta)$, etc.

Note: You don't always need to find the maximizer explicitly! Sometimes you may stop at things like $\cos(\theta) = \frac{1}{3}$ and then maximize your function using this info (see problems 6 and the review session on Monday evening)

Problem 1

[4.7.16] What is the largest possible area of a rectangle with perimeter 16?

Problem 2

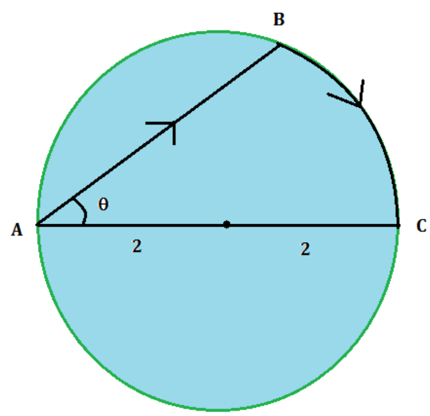
[4.7.18] Find the point on the line $6x + y = 9$ that is closest to the point $(-3, 1)$

Problem 3

[4.7.22] Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Problem 4

[4.7.46] **[HARD]** A woman at a point A on the shore of a circular lake with radius $2mi$ wants to arrive at the point C diametrically opposite A on the other side of the lake in the shortest possible time. She can walk at the rate of $4mph$ and row a boat at $2mph$. How should she proceed?



1A/Handouts/Shore.png

Problem 5

[4.7.52] At which points on the curve $y = 1 + 40x^3 - 3x^5$ does the tangent line have largest slope?

Problem 6

[4.7.40] Suppose $F(\theta) = \frac{\mu W}{\mu \sin(\theta) + \cos(\theta)}$. Show that F is maximized if $\tan(\theta) = \mu$