Math 1A - Concavity

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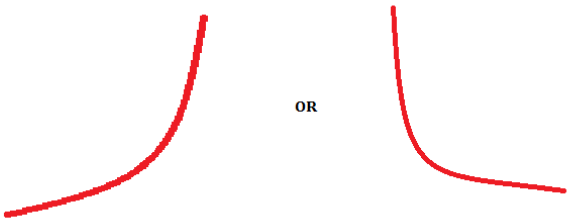
1 Concavity

We've learned last time that f' gives us a lot of valuable information about the graph of f. Now, f'' gives us even more info about the graph of f, namely about **concavity**.

1A/Handouts/Concave up.png

Definition: f is concave up if it looks like this:

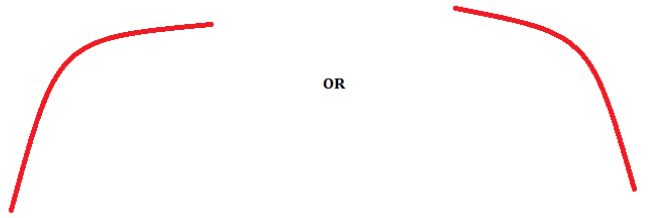


Increasing and Concave up

Decreasing and Concave up

1A/Handouts/Concave down.png

Definition: f is concave down if it looks like this:

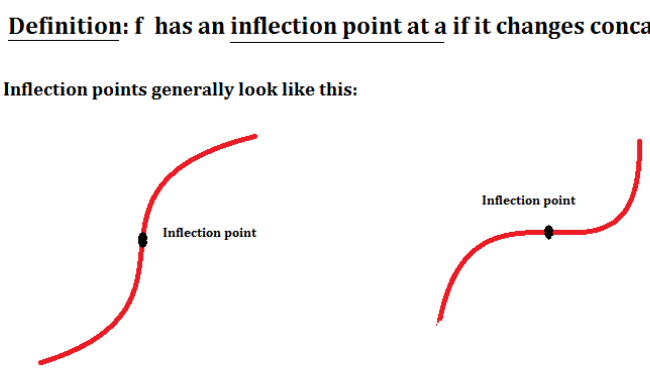


Increasing and Concave down

OR

Decreasing and Concave down

1A/Handouts/Inflection point.png



Mnemonic device: Concave up like a cup, Concave down like a frown!

2 Concavity test

There's an easy test to determine whether a function is concave up or concave down!

Concavity Test.
- *If* $f''(x) > 0$ *on* (a, b)*, then* f *is concave up on* (a, b)
- *If* $f''(x) < 0$ *on* (a, b)*, then* f *is concave down on* (a, b)

Problem. *Determine where* $f(x) = 2x^3 + 3x^2 - 36x$ *is concave up, concave down, and where it has inflection points.*

$f''(x) = 12x + 6$. It is easy to see that $f''(x) < 0$ when $x < -\frac{1}{2}$, $f''(x) = 0$ when $x = -\frac{1}{2}$ and $f''(x) > 0$ when $x > -\frac{1}{2}$. Hence f is concave down on $\boxed{(-\infty, -\frac{1}{2})}$, concave up on $\boxed{(-\frac{1}{2}, \infty)}$ and it has an inflection point at $\boxed{x = -\frac{1}{2}}$ (it changes concavity there!)

3 Graphing

Now we have a bunch of info about f (we can determine where it's increasing and decreasing, we can find its local maximums and minimums, and we can see where it's concave up and down). How can we use this to sketch a rough graph of f ?

Problem. *Given that the sign tables for f look like this, graph f*

1A/Handouts/Sign Tables.png

x	-oo		-3		3		oo
f'(x)		—	0	+	0	—	
f(x)	oo ↘		-5	↗	5	↘	-oo

x	-oo		-1		0		1		oo
f'(x)		+	0	—	0	+	0	—	
f(x)		C.U.	I.P. = (-1,0)	C.D.	I.P. = (0,1)	C.U.	I.P. = (1,2)	C.D.	

The idea is: Begin by marking the important points (i.e. the points on the first row of your sign tables, here $-3, -1, 0, 1, 3$) and then draw the graph by using your gut feeling!

1A/Handouts/Pre-graph.png