MATH 1A - SOLUTION TO 4.2.36

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- **Problem:** A number a is called a **fixed point** of a function f if $f(a) = a$ (for example, 1 is a fixed point of $f(x) = x^2$. Prove that if $f'(x) \neq 1$ for all x, then f has at most one fixed point.

- **Solution:**

This is again a proof by contradiction!

Suppose f has (at least) two fixed points a and b.

Then, by definition of a fixed point, $f(a) = a$, and $f(b) = b$.

However, by the **Mean Value Theorem**, there is a c in (a, b) such that:

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Now using the fact that $f(b) = b$ and $f(a) = a$, we get:

$$\frac{b - a}{b - a} = f'(c)$$

So

$$1 = f'(c)$$

That is, $f'(c) = 1$. However, by assumption, $f'(x) \neq 1$ for all x, so in particular setting $x = c$ gives $f'(c) \neq 1$.

but since $f'(c) = 1$, we get $1 \neq 1$, which is a contradiction!

Hence f has at most one fixed point!

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