1) Evaluate \( f \) at the endpoints 0 and \( \frac{\pi}{2} \)

\[
f(0) = 2 + 0 = 2, \quad f\left(\frac{\pi}{2}\right) = 0 + 0 = 0
\]

2) Find the critical numbers of \( f \)

\[
f'(t) = -2 \sin(t) + 2 \cos(2t)
\]

\[
f'(t) = 0 \\
-2 \sin(t) + 2 \cos(2t) = 0 \\
\sin(t) = \cos(2t)
\]

Here comes the tricky part! This seems impossible to solve, but ideally we’d like to write the right-hand-side just in terms of \( \sin(x) \) in order to have a shot at solving this!

Start with \( \cos(2t) = \cos^2(t) - \sin^2(t) \) (the double-angle formula for \( \cos \)).

Moreover \( \cos^2(t) = 1 - \sin^2(t) \) (because \( \cos^2(t) + \sin^2(t) = 1 \))

So we get \( \cos(2t) = 1 - \sin^2(t) - \sin^2(t) = 1 - 2\sin^2(t) \). So our original equation becomes:

\[
\sin(t) = 1 - 2\sin^2(t)
\]

which you can rewrite as \( 2\sin^2(t) + \sin(t) - 1 = 0 \).

This again looks impossible to solve, but notice that this is just a quadratic equation in \( \sin(t) \)!

So let \( X = \sin(t) \), then we get:

\[
2X^2 + X - 1 = 0
\]

And using the quadratic formula (or your factoring skills), we get:

\[
(2X - 1)(X + 1) = 0
\]

So \( X = \frac{1}{2} \) or \( X = -1 \). That is, \( \sin(t) = \frac{1}{2} \) or \( \sin(t) = -1 \).

However, remember that we’re only focusing on \([0, \frac{\pi}{2}]\), so in particular \( \sin(t) = \frac{1}{2} \) has only one solution in \([0, \frac{\pi}{2}]\), namely \( t = \frac{\pi}{6} \), and \( \sin(t) = -1 \) has NO solution in \([0, \frac{\pi}{2}]\).
It follows that the only critical number of $f$ in $[0, \frac{\pi}{2}]$ is $t = \frac{\pi}{6}$ (there are no numbers where $f$ is not differentiable, so in fact those are all the critical numbers).

And we get $f(\frac{\pi}{6}) = \sqrt{3} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

3) Compare all the candidates you have:

Our candidates are $f(0) = 2$, $f(\frac{\pi}{2}) = 0$ and $f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2} \approx 2.59$.

Hence, the absolute minimum of $f$ on $[0, \frac{\pi}{2}]$ is $f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2}$ (the smallest candidate), and the absolute maximum of $f$ on $[0, \frac{\pi}{2}]$ is $f(\frac{\pi}{2}) = 0$ (the largest candidate)