MATH 1A - SOLUTION TO 4.1.57

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1) Evaluate f at the endpoints 0 and $\frac{\pi}{2}$

    $f(0) = 2 + 0 = 2$, $f(\frac{\pi}{2}) = 0 + 0 = 0$

2) Find the critical numbers of f

    $f'(t) = -2\sin(t) + 2\cos(2t)$

$$\begin{aligned} f'(t) &= 0 \\ -2\sin(t) + 2\cos(2t) &= 0 \\ \sin(t) &= \cos(2t) \end{aligned}$$

Here comes the tricky part! This seems impossible to solve, but ideally we'd like to write the right-hand-side just in terms of $\sin(x)$ in order to have a shot at solving this!

Start with $\cos(2t) = \cos^2(t) - \sin^2(t)$ (the double-angle formula for $\cos$).
Moreover $\cos^2(t) = 1 - \sin^2(t)$ (because $\cos^2(t) + \sin^2(t) = 1$)
So we get $\cos(2t) = 1 - \sin^2(t) - \sin^2(t) = 1 - 2\sin^2(t)$. So our original equation becomes:

$$\sin(t) = 1 - 2\sin^2(t)$$

which you can rewrite as $2\sin^2(t) + \sin(t) - 1 = 0$.

This again looks impossible to solve, but notice that this is just a quadratic equation in $\sin(t)$! So let $X = \sin(t)$, then we get:

$$2X^2 + X - 1 = 0$$

And using the quadratic formula (or your factoring skills), we get:

$$(2X - 1)(X + 1) = 0$$

So $X = \frac{1}{2}$ or $X = -1$. That is, $\sin(t) = \frac{1}{2}$ or $\sin(t) = -1$.
HOWEVER, remember that we're only focusing on $[0, \frac{\pi}{2}]$, so in particular $\sin(t) = \frac{1}{2}$ has only one solution in $[0, \frac{\pi}{2}]$, namely $t = \frac{\pi}{6}$, and $\sin(t) = -1$ has **NO** solution in $[0, \frac{\pi}{2}]$.

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It follows that the only critical number of f in $[0, \frac{\pi}{2}]$ is $\boxed{t = \frac{\pi}{6}}$ (there are no numbers where f is not differentiable, so in fact those are all the critical numbers).

And we get $f(\frac{\pi}{6}) = \sqrt{3} + \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$

3) Compare all the candidates you have:

Our candidates are $f(0) = 2$, $f(\frac{\pi}{2}) = 0$ and $f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2} \approx 2.59$.

Hence, the absolute minimum of f on $[0, \frac{\pi}{2}]$ is $\boxed{f(\frac{\pi}{2}) = 0}$ (the smallest candidate), and the absolute maximum of f on $[0, \frac{\pi}{2}]$ is $\boxed{f(\frac{\pi}{6}) = \frac{3\sqrt{3}}{2}}$ (the largest candidate)