As is usual for related rates problems, let’s draw a picture:

Let $\theta$ be the angle between the hour hand and the minute hand. Now, what we want to calculate is $D'(\theta)$, where the $'$ indicates differentiation with respect to the time variable.

How can we relate $D(\theta)$ with what we know? This is easy! We know an angle $\theta$ and the lengths of $AB$ and $AC$ in the picture, so let’s just use the law of cosines. We get:

$$BC^2 = AC^2 + AB^2 - 2 \cdot AC \cdot AB \cdot \cos(\theta)$$

That is:
\[ D(\theta)^2 = 8^2 + 4^2 - 2 \cdot 8 \cdot 4 \cdot \cos(\theta) \]

Which you can write as:

\[ D(\theta)^2 = 80 - 64 \cos(\theta) \]

Now differentiate with respect to time!

We get:

\[ 2D(\theta)D'(\theta) = 64 \sin(\theta) \frac{d\theta}{dt} \]

And now, all we need to do is to plug in everything we know!

First of all \( \theta = \frac{2\pi}{12} = \frac{\pi}{6} \) (basically, the whole circle corresponds to 2\(\pi\), and so \(\frac{1}{12}\) of the circle corresponds to \(\frac{2\pi}{12}\)).

In particular, \( \sin(\theta) = \sin(\frac{\pi}{6}) = \frac{1}{2} \).

Now, for \(d(\theta)\), we use the law of cosines again, this time with the value \(\theta = \frac{\pi}{6}\):

\[ D\left(\frac{\pi}{6}\right)^2 = 80 - 64 \cdot \cos\left(\frac{\pi}{6}\right) = 80 - 64 \cdot \frac{\sqrt{3}}{2} = 80 - 32\sqrt{3} \]

So, taking square roots, we get

\[ d\left(\frac{\pi}{6}\right) = \sqrt{80 - 32\sqrt{3}} = 4\sqrt{5 - 2\sqrt{3}} \text{ mm} \]

Finally, we need to compute \(\frac{d\theta}{dt}\). But think about it! In 12 hours, \(\theta = 2\pi\), so the speed of \(\theta\) should be \(\frac{2\pi}{12} = \frac{\pi}{6} = -\frac{11\pi}{6}\) rad/h (notice that \(\theta\) is decreasing, so we wanted a negative answer!)

Finally, we have all our information to get our final answer:

\[ 2D(\theta)D'(\theta) = 64 \sin(\theta) \frac{d\theta}{dt} \]

\[ 2 \cdot (4\sqrt{5 - 2\sqrt{3}}) \cdot D'\left(\frac{\pi}{6}\right) = 64 \cdot \frac{1}{2} \cdot \frac{-11\pi}{6} \]

\[ 2 \cdot (4\sqrt{5 - 2\sqrt{3}}) \cdot D'\left(\frac{\pi}{6}\right) = 32 \cdot \frac{-11\pi}{6} \]

\[ D'\left(\frac{\pi}{6}\right) = 16 \cdot \frac{-11\pi}{6} \cdot \frac{4\sqrt{5 - 2\sqrt{3}}}{\sqrt{5 - 2\sqrt{3}}} \]

\[ D'\left(\frac{\pi}{6}\right) = 4 \cdot \frac{-11\pi}{6} \cdot \frac{\sqrt{5 - 2\sqrt{3}}}{\sqrt{5 - 2\sqrt{3}}} \]

\[ D'\left(\frac{\pi}{6}\right) = -\frac{22\pi}{3\sqrt{5 - 2\sqrt{3}}} \]

So our final answer is

\[ D'\left(\frac{\pi}{6}\right) = -\frac{22\pi}{3\sqrt{5 - 2\sqrt{3}}} \text{ mm/h} \approx -18.55 \text{ mm/h} \]