1) As usual, let’s draw a picture of the situation:

Here, $x$ is the distance between the runner and the friend, and $s$ is the length of the arc corresponding to $\theta$.

2) We want to figure out $\frac{dx}{dt}$ when $x = 200$

3) Looking at the picture, it looks like we should use the law of cosines (because we have info about $\theta$ and about 2 of the 3 sides of the triangle)
\[ x^2 = 100^2 + 200^2 - 2(100)(200) \cos(\theta) \]

In other words:
\[ x^2 = 50000 - 40000 \cos(\theta) \]

4) Hence \( 2x \frac{dx}{dt} = 40000 \sin(\theta) \frac{d\theta}{dt} \)

5) First of all \( x = 200 \) in this case. Also, by definition of a radian, we know that \( s = 100\theta \), whence \( \frac{ds}{dt} = 100 \frac{d\theta}{dt} \). But we’re given that \( \frac{ds}{dt} = 7 \text{ m/s} \), so \( \frac{d\theta}{dt} = \frac{7}{100} = 0.07 \).

So all we got to figure out is \( \sin(\theta) \)

For this, draw the same picture as above, except you let \( x = 200 \). And in this case, we use the law of cosines again:
\[ 200^2 = 100^2 + 200^2 - 2(100)(200) \cos(\theta) \]
\[ = -10000 = -40000 \cos(\theta) \]
\[ \cos(\theta) = \frac{1}{4} \]

Now you can use either the triangle method to figure out what \( \sin(\theta) \) is (all you gotta do is calculate \( \sin(\cos^{-1}(\frac{1}{4})) \), or, even easier, notice that \( \sin(\theta) = \sqrt{1 - \cos^2(\theta)} \) (this works because \( \sin(\theta) > 0 \) because we assume that \( \theta \) is between 0 and \( \frac{\pi}{2} \). Hence \( \sin(\theta) = \sqrt{1 - \frac{1}{16}} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4} \).

PHEW!!! Now we have all the info we need to solve the problem:
\[ 2x \frac{dx}{dt} = 40000 \sin(\theta) \frac{d\theta}{dt} \]
\[ 2(200) \frac{dx}{dt} = 40000 \left( \frac{\sqrt{15}}{4} \right)(0.07) \]
\[ 400 \frac{dx}{dt} = 700 \sqrt{15} \]
\[ \frac{dx}{dt} = \frac{7}{4} \sqrt{15} \]

Hence \( \frac{dx}{dt} = \frac{7}{4} \sqrt{15} \)