MATH 1A - SOLUTION TO 3.9.43

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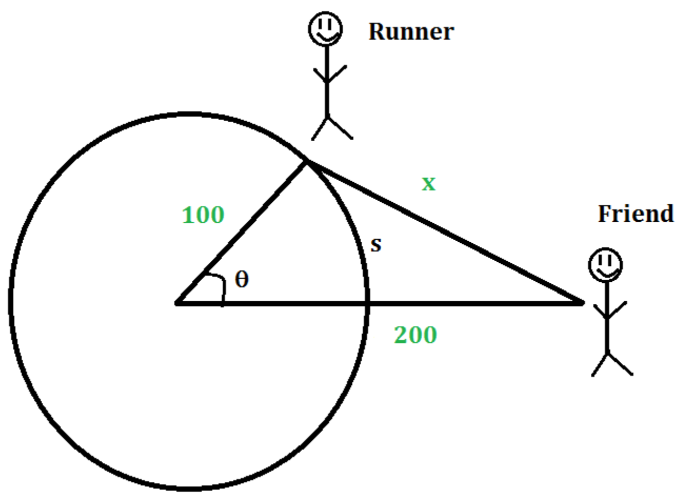
1) As usual, let's draw a picture of the situation:

1A/Solutions/Runners.png



Here, x is the distance between the runner and the friend, and s is the length of the arc corresponding to θ

2) We want to figure out $\frac{dx}{dt}$ when $x = 200$
3) Looking at the picture, it looks like we should use the law of cosines (because we have info about θ and about 2 of the 3 sides of the triangle)

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$$x^2 = 100^2 + 200^2 - 2(100)(200)\cos(\theta)$$

In other words:

$$x^2 = 50000 - 40000\cos(\theta)$$

4) Hence $2x\frac{dx}{dt} = 40000\sin(\theta)\frac{d\theta}{dt}$

5) First of all $x = 200$ in this case. Also, by definition of a radian, we know that $s = 100\theta$, whence $\frac{ds}{dt} = 100\frac{d\theta}{dt}$. But we're given that $\frac{ds}{dt} = 7m/s$, so $\frac{d\theta}{dt} = \frac{7}{100} = 0.07$.

So all we got to figure out is $\sin(\theta)$

For this, draw the same picture as above, except you let $x = 200$. And in this case, we use the law of cosines **again**:

$$200^2 = 100^2 + 200^2 - 2(100)(200)\cos(\theta)$$

$$-10000 = -40000\cos(\theta)$$

$$\cos(\theta) = \frac{1}{4}$$

Now you can use either the triangle method to figure out what $\sin(\theta)$ is (all you gotta do is calculate $\sin(\cos^{-1}(\frac{1}{4}))$, or, even easier, notice that $\sin(\theta) = \sqrt{1-\cos^2(\theta)}$ (this works because $\sin(\theta) > 0$ because we assume that θ is between 0 and $\frac{\pi}{2}$. Hence $\sin(\theta) = \sqrt{1-\frac{1}{16}} = \sqrt{\frac{15}{16}} = \frac{\sqrt{15}}{4}$.

PHEW!!! Now we have all the info we need to solve the problem:

$$\begin{aligned}
2x\frac{dx}{dt} &= 40000\sin(\theta)\frac{d\theta}{dt} \\
2(200)\frac{dx}{dt} &= 40000(\frac{\sqrt{15}}{4})(0.07) \\
400\frac{dx}{dt} &= 700\sqrt{15} \\
\frac{dx}{dt} &= \frac{7}{4}\sqrt{15}
\end{aligned}$$

Hence $\boxed{\frac{dx}{dt} = \frac{7}{4}\sqrt{15}}$