MATH 1A - SOLUTION TO 3.9.38

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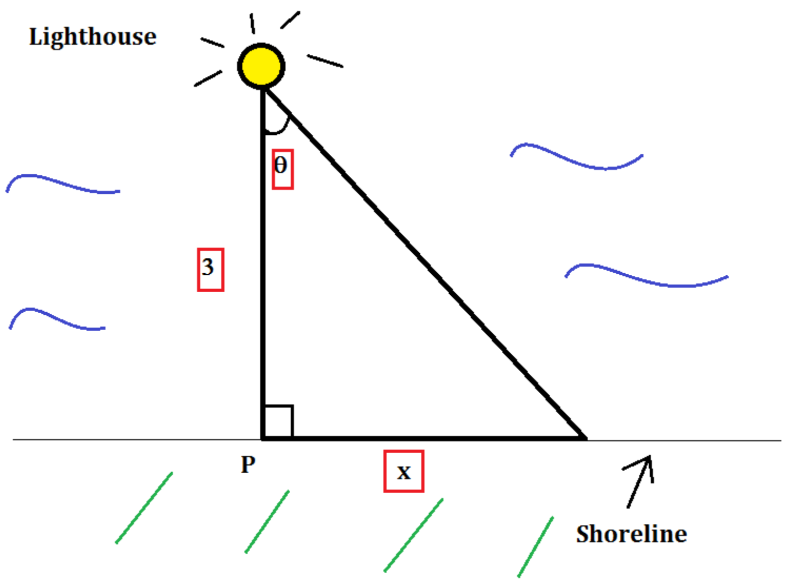
1) Again, draw a picture of the situation:

1A/Solutions/Lighthouse.png



Here, x is the distance between P and the beam of light.

2) We want to figure out $\frac{dx}{dt}$ when $x = 1$

3) Looking at the picture, because we have info about the derivative of θ (see below), we use the definition of $\tan(\theta)$:

$$\tan(\theta) = \frac{x}{3}$$

So $x = 3\tan(\theta)$

Date: Wednesday, March 9th, 2011.

4) Hence $\frac{dx}{dt} = 3\sec^2(\theta)\frac{d\theta}{dt}$
5) First of all, we actually know what $\frac{d\theta}{dt}$ is! Since the lighthouse makes 4 revolutions per minute and one revolution corresponds to 2π, we know that $\frac{d\theta}{dt} = -8\pi$ (think of speed = distance/time, and here time = 1 minute, and 'distance' = 8π, also you put a minus-sign since x is decreasing!).

   Moreover, by drawing the **exact** same picture as above, except with $x = 1$, we can calculate $\sec^2(\theta)$, namely:

$$\sec(\theta) = \frac{hypothenuse}{adjacent} = \frac{\sqrt{10}}{3}$$

   And the $\sqrt{10}$ we get from the Pythagorean theorem!
   It follows that $\sec^2(\theta) = \left(\frac{\sqrt{10}}{3}\right)^2 = \frac{10}{9}$.
   Now we got all of the info we need to conclude the problem:

$$\begin{aligned} \frac{dx}{dt} =& 3\sec^2(\theta)\frac{d\theta}{dt} \\ \frac{dx}{dt} =& 3(\frac{10}{9})(-8\pi) \\ \frac{dx}{dt} =& -\frac{240\pi}{9} \\ \frac{dx}{dt} =& -\frac{80\pi}{3} \end{aligned}$$

   Whence $\boxed{\frac{dx}{dt} = -\frac{80\pi}{3} rad/min}$