MATH 1A - SOLUTION TO 3.8.11

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The problem asks about radioactive decay, so as usual, we have $y' = ky$, so $y(t) = Ce^{kt}$. Now we're given two things: First of all, the half-life is $t = 5730$ years, so $y(5730) = \frac{y(0)}{2} = \frac{C}{2}$. Moreover, we know that at a certain time t^* (we **want to find** t^*), $y(t^*) = 0.74y(0) = 0.74C$. Now even though we don't know what C is, we can still solve for t^*.

The following calculation helps us find k:

$$\begin{aligned} y(5730) &= \frac{C}{2} \\ Ce^{5730k} &= \frac{C}{2} \\ e^{5730k} &= \frac{1}{2} \\ 5730k &= \ln(\frac{1}{2}) \\ k &= \frac{\ln(\frac{1}{2})}{5730} \end{aligned}$$

Whence $y(t) = Ce^{\frac{\ln(\frac{1}{2})}{5730}t} = C(\frac{1}{2})^{\frac{t}{5730}}$

Now we're given that $y(t^*) = 0.74C$, and the following calculation helps us solve for t^*:

$$\begin{aligned} y(t^*) &= 0.74C \\ C(\frac{1}{2})^{\frac{t^*}{5730}} &= 0.74C \\ (\frac{1}{2})^{\frac{t^*}{5730}} &= 0.74 \\ \frac{t^*}{5730}\ln(\frac{1}{2}) &= \ln(0.74) \\ t^* &= 5730\frac{\ln(0.74)}{\ln(\frac{1}{2})} \\ t^* &\approx 2489 \end{aligned}$$

So $\boxed{t^* \approx 2489 \text{ years}}$ (notice how we didn't even need info about C to figure this out!)

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