MATH 1A - SOLUTION TO 3.7.24

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First of all, we know two things, namely $f(0) = 20$ and $f'(0) = 12$.

But by the chain rule:

$$f'(t) = -0.7be^{-0.7t} \frac{-a}{(1 + be^{-0.7t})^2} = \frac{0.7abe^{-0.7t}}{(1 + be^{-0.7t})^2}$$

So from $f(0) = 20$, we get:

$$\frac{a}{1+b} = 20$$

And from $f'(0) = 12$, we get:

$$\frac{0.7ab}{(1+b)^2} = 12$$

From $\frac{a}{1+b} = 20$, we get $a = 20(1 + b)$, and plugging this into the second equation, we get:

$$\begin{aligned}
\frac{(0.7)(20)(1+b)b}{(1+b)^2} &= 12 \\
\frac{14b}{1+b} &= 12 \\
14b &= 12(1+b) \\
14b &= 12 + 12b \\
2b &= 12 \\
b &= 6
\end{aligned}$$

And so $a = 20(1 + 6) = 20(7) = 140$.

Therefore, we have $\boxed{a = 140}$ and $\boxed{b = 6}$.

Finally, to find out what happens in the long run, we need to calculate $\lim_{t\to\infty} f(t)$. But notice that $\lim_{t\to\infty} e^{-0.7t} = 0$, and so $\lim_{t\to\infty} f(t) = \frac{a}{1+0} = a = \boxed{140}$.

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