

Applications of derivatives (in real life!)

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Chemistry

Problem 1

[That should look familiar!] At time t_0 , a beaker contains 2 grams of salt dissolved in 5 ounces of water. At time t_0 , water is being added at 10 ounces/min and salt is being added at 3 grams/min. How fast is the concentration of salt (in grams per ounces) changing at t_0 ?

Physics

Problem 2

Suppose the position of a certain particle on the real line is given by

$$s(t) = t^3 - 12t^2 + 36t$$

When the acceleration of the particle is 0, is the particle moving to the left or to the right?

Problem 3

Newton's law of gravitation says that the magnitude F of the force exerted by a body of mass m on the body of mass M is:

$$F = \frac{GmM}{r^2}$$

where G is the gravitational constant and r is the distance between the two bodies.

- (a) Find $\frac{dF}{dr}$ and explain its meaning. What does the $-$ sign indicate?
- (b) Suppose you know that the earth attracts an object with a force that decreases at 2 N/km where $r = 20000$ km. How fast does this force change when $r = 10000$ km?

Biology

Problem 4

The velocity of blood in an artery of length ℓ and radius R is given by Poiseuille's law:

$$v(r) = \frac{P}{4\nu\ell} (R^2 - r^2)$$

where r is the distance to the central axis of the artery (see picture in section), P is the blood pressure and ν is the viscosity of the blood (which is higher if there is more cholesterol).

Suppose that you're at risk of heart disease if the absolute value of the **velocity gradient** (rate of change of velocity with respect to r) is less than $100 \times \frac{R}{2}$.

Suppose you're treating a patient whose vessel radius $R = 0.01$ cm, whose length $\ell = 3$ cm, blood pressure $P = 3000$ and blood viscosity is $\nu = 0.027$.

Is that patient at risk for heart disease?

Economics

Problem 5

Assume Peyam's happiness/utility function is given by:

$$U(c) = c + 2\sqrt{c} + 100$$

where c is the number of chocolate cakes consumed.

- (a) What does $U(0)$ represent?
- (b) Find $U'(c)$. What does it represent? Why is it positive?
- (c) Explain why $U''(c) < 0$

Problem 6

Suppose you're the CEO of Pigh-am-Tech, and suppose you're trying to decide whether to invest 1 million dollars in machines or not. Suppose that when you spend the 1 million dollars, you will obtain 500 machines and that number is increasing by 1.1 machines per dollar spent. Moreover, suppose that the marginal profit of having 500 machines is 0.9 dollars per machine.

Should you or should you not make that investment?