DEFINITIONS, THEOREMS, AND PROOFS FOR FINAL EXAM

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Note: The number in parentheses refers to the lecture note where the corresponding topic is adressed!
Note: You don't need to memorize **all** of those definitions and theorems (although some of them are important, written in **boldface**). It is equally important to understand 'what's going on' in those theorems.

1. Definitions and Theorems

- Function (1)
- Domain/Range of a function (1)
- Vertical Line Test (1)
- Absolute Value Function (1)
- Increasing/Decreasing Functions (1)
- Composition of Functions, i.e. $f \circ g$ (2)
- One-to-one (2)
- Horizontal Line Test (2)
- f^{-1} (2)
- $\log_a(x)$ (3)
- e, and e^x (3)
- $\ln$, and $\ln_x$ (3)
- Secant line of f going through $(a, f(a))$ and $(b, f(b))$ (3)
- Direct Substitution Property (4)
- Squeeze Theorem (5)
- $\lim_{x\to a} f(x) = L$ (5, or see text page 110)
- $\lim_{x\to a^+} f(x) = L$ (7)
- $\lim_{x\to a^-} f(x) = L$ (7)
- $\lim_{x\to a} f(x) = \infty$ (7)
- $\lim_{x\to a} f(x) = -\infty$ (7)
- Continuity (7)
- **Intermediate Value Theorem** (7)
- $\lim_{x\to\infty} f(x) = L$ (8, see text page 138)
- $\lim_{x\to-\infty} f(x) = L$ (8, see text page 138)
- Horizontal Asymptotes (8)
- $f'(a)$ (9)
- Tangent line to the graph of f at a (9)
- f' (9)
- f'' (10)
- e (the 'derivative' definition) (12)
- The sum rule for derivatives (12)
- The power rule for derivatives (12)

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- The product rule for derivatives (12)
- The quotient rule for derivatives (13)
- The Chain Rule (14)
- e (the 'limit definition') (16)
- Newton's law of cooling (17, see text page 237)
- Linear approximation of f at a (19)
- f has an absolute maximum/minimum at a (19)
- f has a local maximum/minimum at a (20)
- **Extreme Value Theorem** (20)
- Fermat's Theorem (20)
- Critical Number (20)
- Rolle's Theorem (21)
- **Mean Value Theorem** (21)
- Identity principle for derivatives (21)
- Increasing/Decreasing Test (22)
- First Derivative Test (22)
- Concave Upward/Downward (23)
- Concavity Test (23)
- Second Derivative Test (23)
- Inflection Point (23)
- L'Hopital's Rule (24)
- Slant Asymptotes (25)
- Newton's Method (28)
- F is an antiderivative of f (29)
- x_i (31)
- Left-hand-sum L_n (31)
- Right-hand-sum R_n (31)
- Sample points x_i^* (31)
- Riemann Sums (32)
- Definite Integral $\int_a^b f(x)dx$ (rigorous and non-rigorous definition) (32)
- Continuous functions (or functions with finitely many jump discontinuitites) on $[a, b]$ are Riemann integrable (32)
- Comparison Inequality (33)
- **Fundamental Theorem of Calculus, Part I** (33)
- **Fundamental Theorem of Calculus, Part II** (34)
- Indefinite Integral $\int f(x)dx$ (36)
- The Substitution Rule (36)
- Area between two curves (38)
- Volume (39, or see text page 423)
- Disk and Washer Methods (39)
- Shell Method (40)
- Average Value of a function (40)
- Work (41)
- **Mean Value Theorem for Integrals** (41)
- Average velocity - 2 definitions (41)

2. PROOFS

- Define 2^x, or, in general, a^x (1)
- Differentiability implies continuity (12)
- The Product Rule (12)
- The Quotient Rule, using the Product Rule (13)
- $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$ (13, 14)
- $\lim_{x\to 0} \frac{1-\cos(x)}{x} = 0$, using the above fact (13)
- $\frac{d}{dx} \sin(x) = \cos(x)$ (13)
- $\frac{d}{dx} \cos(x) = -\sin(x)$ (13)
- $\frac{d}{dx} \ln(x) = \frac{1}{x}$ (14)
- $\frac{d}{dx} \sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$ (15)
- $\frac{d}{dx} \tan^{-1}(x) = \frac{1}{1+x^2}$ (16)
- $\lim_{x\to 0} (1+x)^{\frac{1}{x}} = e$ (16)
- $\lim_{x\to 0} \frac{\ln(1+x)}{x} = 1$ (16)
- $\frac{d}{dx} (\ln(|x|)) = \frac{1}{x}$ (17)
- The solution of Newton's law of cooling (see page 237 of text)
- $\sin(x) \approx x$ (19)
- $f'(x) = 0$ for every $x \in (a, b)$ implies f constant on (a, b) (21)
- Identity Principle for Derivatives (21)
- f' never 0 implies f is invertible (21)
- Mean Value Theorem (using Rolle's Theorem) (21,22)
- Rolle's Theorem (21)
- Fundamental Theorem of Calculus, Part I (35)
- FTC, Part II, **using** FTC Part I (36)
- The Substitution Rule (36)
- If f is odd, $\int_{-a}^{a} f(x)dx = 0$ (37)
- If f is even, $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ (37)
- Area between two curves (38)
- $\int_0^1 \sqrt{1-x^2}dx = \int_0^{\frac{\pi}{2}} \sin^2(\theta)d\theta = \frac{\pi}{4}$ (38)
- Formula for the volume of a sphere (39)
- Formula for the Disk, Washer and Shell Methods (39, 40)
- Formula for Work (41)
- Mean-Value Theorem for Integrals (41)
- $\frac{1}{b-a}\int_a^b v(t)dt = \frac{s(b)-s(a)}{b-a}$ (41)