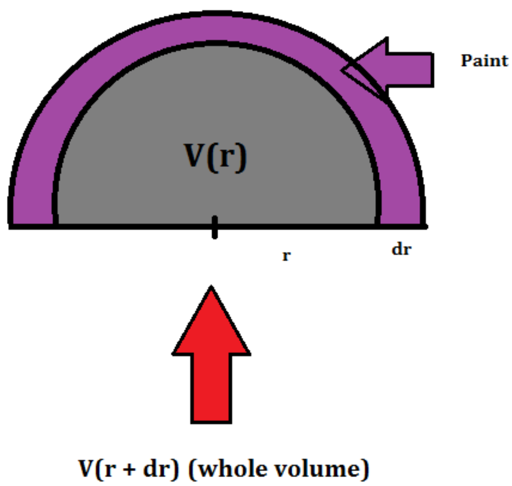
MATH 1A - SOLUTION TO 3.10.36

PEYAM RYAN TABRIZIAN

First of all, let's recall the formula for the **volume** of a sphere of radius r: $VA = \frac{4}{3}\pi r^3$. Now, notice that we're dealing with a **hemisphere**, so the volume should be $V = \frac{2}{3}\pi r^3$ (half of VA). Now, the amount of paint needed to coat the hemisphere is precisely:

$$V(r + dr) - V(r)$$

It's basically the volume of just a little bit more than the hemisphere minus the volume of the hemisphere. It makes more sense if you draw a little picture of what's going on (the real picture should be 3-dimensional, but I used two dimensions to simplify the situation):



1A/Hemisphere.png

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But, now, we want to reformulate this as a linear approximation problem!
So let $x = r + dr$ and $a = r$, and $f(x) = V(x)$. Then, we know from lecture/section that:

$$f(x) \approx L(x)$$

Where:

$$L(x) = f(a) + f'(a) \cdot (x - a)$$

And replacing f, x, a by what we know, we get:

$$L(r + dr) = V(r) + V'(r) \cdot (r + dr - r) = V(r) + V'(r) \cdot dr$$

Now, let's remember that we want to find:

$$V(r + dr) - V(r) \approx L(r + dr) - V(r) = (V(r) + V'(r) \cdot dr) - V(r) = V'(r) \cdot dr$$

So all we need to compute is $V'(r) \cdot dr$. But $V'(r) = \frac{2}{3}\pi \cdot 3 \cdot r^2 = 2\pi r^2$.
Finally, plug in r and dr. We have $r = \frac{50}{2} = 25$m (the radius is half of the diameter), and $dr = 0.05$cm. Now **BEWARE** that the units are not the same, so we'll have to convert for example dr in m.
We get: $dr = 0.0005$ m, and $r + dr = 25.0005$m.
Finally, we get:

$$V(r + dr) - V(r) = V'(r) \cdot dr = 2\pi(25)^2 \cdot 0.0005m^3 \approx 1.96m^3$$

Hence the amount of paint needed is: $\boxed{\approx 1.96\ m^3}$