Vector Spaces

A vector space is a set V together with two operations, vector addition and scalar multiplication, satisfying the following axioms.

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ if $\mathbf{u}$ and $\mathbf{v}$ are in V.

2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$ if $\mathbf{u}, \mathbf{v}$, and $\mathbf{w}$ are in V.

3. There exists a $\mathbf{0}$ in V such that $\mathbf{0} + \mathbf{v} = \mathbf{v}$ for all $\mathbf{v}$ in V.

4. For every $\mathbf{v}$ in V there exists a $-\mathbf{v}$ in V such that $\mathbf{v} + -\mathbf{v} = \mathbf{0}$.

5. $r(\mathbf{u} + \mathbf{v}) = r\mathbf{u} + r\mathbf{v}$ if r is in R and $\mathbf{u}, \mathbf{v}$ are in V.

6. $(r + s)\mathbf{v} = r\mathbf{v} + s\mathbf{v}$ if r, s are in R and $\mathbf{v}$ is in V.

7. $(rs)\mathbf{v} = r(s\mathbf{v})$ if r, s in R and $\mathbf{v}$ is in V.

8. $1\mathbf{v} = \mathbf{v}$ if $\mathbf{v}$ is in V.

Some key concepts

1. Linear subspaces
2. Linear combinations
3. Span
4. Linear independence
5. Basis for a vector space
6. Dimension of a vector space