Linear transformations

Definition: Let V and W be vector spaces. A *linear transformation* from V to W is a function

$$T: V \to W$$

mapping each element of V to an element of W, with the following properties:

- $T(\mathbf{v} + \mathbf{v}') = T(\mathbf{v}) + T(\mathbf{v}')$ for $\mathbf{v}, \mathbf{v}' \in V$.
- $T(a\mathbf{v}) = aT(\mathbf{v})$ for $a \in \mathbf{R}$ and $\mathbf{v} \in V$.

If $A \in M_{mn}$, then the multiplication by A on the left is a linear transformation

$$T_A: M_{n1} \to M_{m1}.$$

If B is a basis for an n-dimensional vector space V, then the map $\mathbf{v} \mapsto [\mathbf{v}]_B$ is a linear transformation

$$V \to \mathbf{R}^n.$$