Renormalized example

The average heartrate in the above examples is 160. Let us define $e := h - 160$, and redo the computations in terms of e ("extra effort") instead of h.

heart rate	extra effort	speed
140	-20	18
150	-10	20
160	0	23
170	10	25
180	20	26
190		

$$s = me + b$$

$$\begin{aligned} 18 &= -20m + b \\ 20 &= -10m + b \\ 23 &= 0m + b \\ 25 &= 10m + b \\ 26 &= 20m + b \end{aligned}$$

$$\begin{pmatrix} 18 \\ 20 \\ 23 \\ 25 \\ 26 \end{pmatrix} = \begin{pmatrix} -20 & 1 \\ -10 & 1 \\ 0 & 1 \\ 10 & 1 \\ 20 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix}$$

This equation $Y = AX$ is again inconsistent. Instead we consider Instead we find $\overline{Y} = A\overline{X}$ with $||Y - \overline{Y}||$ minimal using:

$$A^T\overline{Y} = A^TY = A^TA\overline{X}$$

Now we get:

$$A^TA = \begin{pmatrix} -20 & -10 & 0 & 10 & 20 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} -20 & 1 \\ -10 & 1 \\ 0 & 1 \\ 10 & 1 \\ 20 & 1 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 1000 & 0 \\ 0 & 5 \end{pmatrix}$$

$$
\begin{aligned}
\overline{X} &= (A^T A)^{-1} A^T Y \\
&= \begin{pmatrix} 1000 & 0 \\ 0 & 5 \end{pmatrix}^{-1} A^T Y \\
&= \begin{pmatrix} \frac{1}{1000} & 0 \\ 0 & \frac{1}{5} \end{pmatrix} \begin{pmatrix} -20 & -10 & 0 & 10 & 20 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} Y \\
&= \begin{pmatrix} .21 \\ 22.1 \end{pmatrix}
\end{aligned}
$$

$$S \sim .21e + 22.1$$

$$S \sim .21(h - 160) + 22.1$$

$$S \sim .21h - 11.2$$

heart rate	speed	prediction	error
140	18	18.2	.2
150	20	20.3	.3
160	23	22.4	-.6
170	25	24.5	- .5
180	26	26.6	.6
190		28.7	
200		30.8	