Example: Speed vs. Heartrate

heart rate	speed
140	18
150	20
160	23
170	25
180	26
190	

Model: $s = mh + b$, for some m, b. These must satisfy:

$$\begin{aligned} 18 &= 140m + b \\ 20 &= 150m + b \\ 23 &= 160m + b \\ 25 &= 170m + b \\ 26 &= 180m + b \end{aligned}$$

$$\begin{pmatrix} 18 \\ 20 \\ 23 \\ 25 \\ 26 \end{pmatrix} = \begin{pmatrix} 140 & 1 \\ 150 & 1 \\ 160 & 1 \\ 170 & 1 \\ 180 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix}$$

This equation $Y = AX$ is inconsistent. Instead we find $\overline{Y} = A\overline{X}$ with $||Y - \overline{Y}||$ minimal using:

$$A^T\overline{Y} = A^TY = A^TA\overline{X}$$

Now:

$$A^TA = \begin{pmatrix} 140 & 150 & 160 & 170 & 180 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 140 & 1 \\ 150 & 1 \\ 160 & 1 \\ 170 & 1 \\ 180 & 1 \end{pmatrix}$$

$$A^TA = \begin{pmatrix} 129000 & 800 \\ 800 & 5 \end{pmatrix}$$

$$
\begin{aligned}
\overline{X} &= (A^T A)^{-1} A^T Y \\
&= \begin{pmatrix} 129000 & 800 \\ 800 & 5 \end{pmatrix}^{-1} A^T Y \\
&= \begin{pmatrix} \frac{1}{1000} & \frac{-4}{25} \\ \frac{-4}{25} & \frac{129}{5} \end{pmatrix} \begin{pmatrix} 140 & 150 & 160 & 170 & 180 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} Y \\
&= \begin{pmatrix} .21 \\ -11.2 \end{pmatrix}
\end{aligned}
$$

Thus $\overline{m} = .21$ and $\overline{b} = -11.2$. The model equation is:

$$s \sim .21h - 11.2$$

heart rate	speed	prediction	error
140	18	18.2	.2
150	20	20.3	.3
160	23	22.4	-.6
170	25	24.5	- .5
180	26	26.6	.6
190		28.7	
200		30.8	