Eigenvectors and eigenvalues

Definition: Let A be a matrix and let $\lambda \in \mathbf{R}$.

$E_\lambda := \{X : AX = \lambda X\}$ is a linear subspace of M_{n1}, called the λ-*eigenspace* of A.

Elements of E_λ are called the λ-*eigenvectors* of A.

A number λ is said to be an *eigenvalue* of A if there is some nonzero λ-eigenvector of A.

Theorem: $E_\lambda = NS(A - \lambda I) = NS(\lambda I - A)$. Hence the following are equivalent:

- λ is an eigenvalue of A.
- $NS(\lambda I - A) \neq \{\mathbf{0}\}$.
- $\lambda I - A$ is singular.
- $\det(\lambda I - A) = 0$.