Determinants

Theorem: For each n, there exists a unique function $\det\colon M_{n\times n} \to \mathbf{R}$ with the following properties

- $\det I = 1$.
- $\det(AB) = \det(A)\det(B)$.
- $\det A = 0$ if and only if A is singular.
- $\det(E_{ij}(\lambda)A) = \det A$ for any $i \neq j$ and any λ.
- $\det(E_{ij}A) = -\det A$ for any $i \neq j$.
- $\det(E_i(\lambda)A) = \lambda \det A$ for any i and any λ.
- $\det A^T = \det A$.

Inductive definition of $\det$

For any i,

$$\det A = \sum_j (-1)^{i+j} a_{ij} M_{ij},$$

where M_{ij} is the determinant of the $(n-1) \times (n-1)$ matrix obtained by deleting the ith row and jth column from A.