

Bases and coordinates

Let $B := (\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n)$ be an ordered basis for a vector space V .

Theorem: Every element of V can be written uniquely as $\mathbf{v} = x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \dots + x_n\mathbf{v}_n$, for some (unique) sequence of numbers x_i .

Definition: If $\mathbf{v} \in V$, the sequence (x_1, \dots, x_n) in \mathbf{R}^n such that $\mathbf{v} = x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \dots + x_n\mathbf{v}_n$ called the sequence of *coordinates* of \mathbf{v} with respect to the basis B .

Notation: $[\mathbf{v}]_B = (x_1, \dots, x_n)$.

Formula: If \mathbf{v} and $\mathbf{v}' \in V$ and $a \in \mathbf{R}$,

$$[\mathbf{v} + \mathbf{v}']_B = [\mathbf{v}]_B + [\mathbf{v}']_B.$$

$$[a\mathbf{v}]_B = a[\mathbf{v}]_B.$$