## Jordan Normal Form

- 1. Let  $A := \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$ . Find an invertible matrix S such that  $A = SBS^{-1}$ , where B is in Joran normal form. Use this to find  $A^5$ .
- 2. Suppose that A, B and N are square matrices, and A = B + N, where BN = NB and  $N^2 = 0$ . Show that for any natural number n,  $A^n = B^n + nB^{n-1}N$ . Show that for any polynomial p, p(A) = p(B) + Np'(B), where p' is the derivative of p.
- 3. With the notation of the previous problem, suppose that B is invertible.
  - (a) Find a formula for  $A^{-1}$  in terms of  $B^{-1}$ .
  - (b) Find a formula for  $\sqrt{A}$  in terms of  $\sqrt{B}$ .
  - (c) Guess a formula for  $e^A$  in terms of  $e^B$ .
- 4. Use problem 3 to compute  $A^{-1}$  and  $\sqrt{A}$  if  $A = \begin{pmatrix} 3 & 2 \\ -2 & -1 \end{pmatrix}$ . Use problem 2 to redo the last part of problem 1.