

Solutions to 3.2

February 6th

Section 3.2

Exercises 1 – 10 : express the given vector in terms of its coordinates.

2. *The vector from $(8, -1, -2, -3)$ to $(9, 4, 7, 3)$.*

The vector is $(9, 4, 7, 3) - (8, -1, -2, -3) = (1, 5, 9, 6)$.

4. *The vector from $(5, -1, 8, -2, 3)$ to $(5, -1, 8, -2, 3)$.*

The vector is $(5, -1, 8, -2, 3) - (5, -1, 8, -2, 3) = (0, 0, 0, 0, 0)$.

Exercises 7-10:

$$\mathbf{u} = \begin{bmatrix} 3 \\ -1 \\ 2 \\ 4 \end{bmatrix} \quad \mathbf{v} = \begin{bmatrix} 4 \\ -1 \\ 5 \\ 8 \end{bmatrix} \quad \mathbf{w} = \begin{bmatrix} 8 \\ -7 \\ -6 \\ 1 \end{bmatrix}.$$

8. $2\mathbf{v} + 3\mathbf{w}$.

$$2\mathbf{v} + 3\mathbf{w} = 2 \begin{bmatrix} 4 \\ -1 \\ 5 \\ 8 \end{bmatrix} + 3 \begin{bmatrix} 8 \\ -7 \\ -6 \\ 1 \end{bmatrix} = \begin{bmatrix} 8 \\ -2 \\ 10 \\ 16 \end{bmatrix} + \begin{bmatrix} 24 \\ -21 \\ -18 \\ 3 \end{bmatrix} = \begin{bmatrix} 32 \\ -23 \\ -8 \\ 19 \end{bmatrix}.$$

10. $3\mathbf{v} - 4(5\mathbf{u} - 6\mathbf{w})$.

$$\begin{aligned} 3\mathbf{v} - 4(5\mathbf{u} - 6\mathbf{w}) &= 3 \begin{bmatrix} 4 \\ -1 \\ 5 \\ 8 \end{bmatrix} - 4 \left(5 \begin{bmatrix} 3 \\ -1 \\ 2 \\ 4 \end{bmatrix} - 6 \begin{bmatrix} 8 \\ -7 \\ -6 \\ 1 \end{bmatrix} \right) = \begin{bmatrix} 12 \\ -3 \\ 15 \\ 24 \end{bmatrix} - 4 \left(\begin{bmatrix} 15 \\ -5 \\ 10 \\ 20 \end{bmatrix} - \begin{bmatrix} 48 \\ -42 \\ -36 \\ 6 \end{bmatrix} \right) = \\ &= \begin{bmatrix} 12 \\ -3 \\ 15 \\ 24 \end{bmatrix} - 4 \begin{bmatrix} -33 \\ 37 \\ 46 \\ 14 \end{bmatrix} = \begin{bmatrix} 12 \\ -3 \\ 15 \\ 24 \end{bmatrix} - \begin{bmatrix} -132 \\ 148 \\ 184 \\ 56 \end{bmatrix} = \begin{bmatrix} 144 \\ -151 \\ -169 \\ -32 \end{bmatrix}. \end{aligned}$$

12. *Find the distance between the given points.*

$$[5 \ 1 \ 8 \ -1 \ 2 \ 9], [4 \ 1 \ 4 \ 3 \ 2 \ 8].$$

The distance is

$$\sqrt{(5-4)^2 + (1-1)^2 + (8-4)^2 + (-1-3)^2 + (2-2)^2 + (9-8)^2} = \sqrt{34}.$$

16. Find the norm $\left\| \begin{bmatrix} 8 \\ -1 \\ 2 \\ 5 \end{bmatrix} \right\|$

The norm is $\sqrt{8^2 + (-1)^2 + 2^2 + 5^2} = \sqrt{94}$.

18. Find $\|\mathbf{u}\| + \|\mathbf{v}\|, \|\mathbf{u} + \mathbf{v}\|$ for $\mathbf{u} = (3, -1, -2, 1, 4), \mathbf{v} = (1, 1, 1, 1, 1)$

$$\|\mathbf{u}\| + \|\mathbf{v}\| = \sqrt{3^2 + (-1)^2 + (-2)^2 + 1^2 + 4^2} + \sqrt{1^2 + 1^2 + 1^2 + 1^2 + 1^2} = \sqrt{31} + \sqrt{5}.$$

$$\|\mathbf{u} + \mathbf{v}\| = \|(4, 0, -1, 2, 5)\| = \sqrt{4^2 + 0^2 + (-1)^2 + 2^2 + 5^2} = \sqrt{46}.$$