Math 54 Spring 2005

Solutions to Homework Section 4.4

March 7, 2005

For the following, find the line that best fits the given points by first translating the x-coordinates so that in the resulting system, $\mathbf{Ax} = \mathbf{b}$*,* $\mathbf{A}$ *has orthogonal columns. Then solve the problem, but be sure to translate back to the original coordinates.*

11. $P_1 = (1,2), P_2 = (2,4), P_3 = (3,5)$. Suppose we translate all the x-coordinates to the right by a. It is clear that $mx + b = y$ will give a least-squares solution for these points iff $mx + b' = y$ is a least-squares solution for the translated points, where $b' = b - ma$. The resulting system for this translation is

$$\begin{bmatrix} (1+a) & 1 \\ (2+a) & 1 \\ (3+a) & 1 \end{bmatrix} \begin{bmatrix} m \\ b' \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}.$$

We now need only pick a so that the columns of $\mathbf{A}$ are orthogonal. The choice, $a = -2$ will do, yielding the system:

$$\begin{bmatrix} -1 & 1 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} m \\ b' \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 5 \end{bmatrix}.$$

In this case, $\mathbf{A}^T\mathbf{A}\bar{x} = \mathbf{A}^T\mathbf{y}$ becomes:

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \bar{x} = \begin{bmatrix} 3 \\ 11 \end{bmatrix},$$

the least-squares solution to which is $\begin{bmatrix} m \\ b' \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{11}{3} \end{bmatrix}$. Thus, using the fact that $b = b' + ma$, the least-squares solution to the original problem is $\begin{bmatrix} m \\ b' \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ (\frac{11}{3} - 3) \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ \frac{2}{3} \end{bmatrix}$.

12. $P_1 = (-1,4), P_2 = (1,3), P_3 = (3,0)$. The reasoning is the same as in Exercise 11. Using the same notation, we set $a = -1$, yielding the inconsistent system

$$\begin{bmatrix} -2 & 1 \\ 0 & 1 \\ 2 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix},$$

which has least-squares solution $\begin{bmatrix} m \\ b' \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{7}{3} \end{bmatrix}$. Translating back, we get $\begin{bmatrix} m \\ b \end{bmatrix} = \begin{bmatrix} -1 \\ \frac{4}{3} \end{bmatrix}$.