Answers to Odd-Numbered Exercises

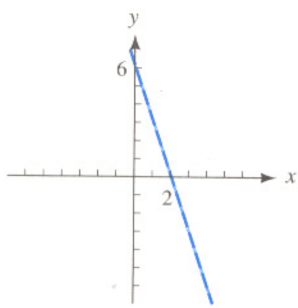
SECTION 1.1

1. Not 3. Linear 5. Not 7. $(x, y) = \left(\frac{5}{2} - 2t, t\right)$ or $(s, \frac{5}{4} - \frac{1}{2}s)$

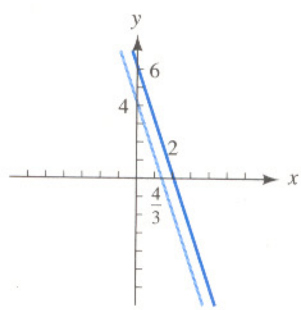
9. $(x, y, z) = (-1 - 2s + \frac{3}{7}t, s, t)$ or $= (s, -\frac{1}{2} - \frac{1}{2}s + \frac{3}{14}t, t)$

11. $(x_1, x_2, x_3, x_4) = (2 + \frac{2}{3}s_1 - \frac{5}{3}s_2 + \frac{1}{3}s_3, s_1, s_2, s_3)$ or $= (t_1, t_2, t_3, 3t_1 - 2t_2 + 5t_3 - 6)$

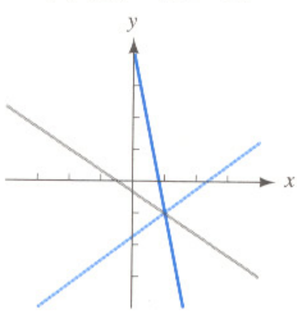
13. Infinitely many



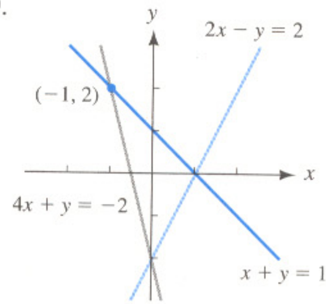
15. None



17. One, $(x, y) = (1, -1)$



19.



Change $2x - y = 2$ to $2x - y = -4$ or to $-4x - y = 2$.

21. $(-1, 2, -2)$ 23. $\left(\frac{7}{8}, -\frac{3}{2}, 0\right)$ 25. $(-\frac{1}{6}, \frac{3}{2}, 1, 2)$ 27. None
29. (a) None (b) $k \neq 4$ (c) 4 31. (a) $k \neq -6$ (b) None (c) -6

SECTION 1.2

1. Y 3. Y 5. N 7. $(0, 3)$ 9. $\left(t, \frac{5}{3}, \frac{1}{2}\right)$
11. $(-2, 1, 3)$ 13. $\left(\frac{1}{2}t - 8, t, -2, 3\right)$ 15. $(3, 2, 1)$
17. $(-2, 1, 3, -1)$ 19. $\left(\frac{3}{2}s - \frac{1}{2}t + \frac{3}{2}, s, 2 - 3t, 5 - 3t, t\right)$
21. Inconsistent 23. $(3, \frac{1}{2}, 2)$ 25. $(-\frac{2}{3}, 1)$ 27. $\left(\frac{55}{16}, \frac{43}{32}\right)$
29. (a) T (b) F (c) T

SECTION 1.3

1. $A = \begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \end{bmatrix}$

3. $\begin{bmatrix} 7 & 6 & 0 \\ -3 & 5 & -3 \end{bmatrix}, \begin{bmatrix} 6 & 8 & 2 \\ -2 & 0 & -12 \end{bmatrix}, \begin{bmatrix} -4 & -2 & 1 \\ 2 & -5 & -3 \end{bmatrix}$

5. $\begin{bmatrix} 10 & 0 & -1 & -3 \\ 3 & 4 & -5 & -6 \end{bmatrix}, \begin{bmatrix} 14 & 4 & 0 & -6 \\ -4 & 0 & -10 & 4 \end{bmatrix}, \begin{bmatrix} -3 & 2 & 1 & 0 \\ -5 & -4 & 0 & 8 \end{bmatrix}$

7. $\begin{bmatrix} 0 & 1 \\ -4 & -1 \end{bmatrix}, \begin{bmatrix} -13 & -20 \\ 8 & 12 \end{bmatrix}$ 9. $\begin{bmatrix} 6 & -10 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 9 & 2 & -9 \\ 0 & 1 & 0 \\ 3 & 2 & -3 \end{bmatrix}$

11. $AB = \begin{bmatrix} 7 & -3 & 1 \\ 6 & 0 & -3 \end{bmatrix}$, BA undefined 13. Both undefined

17. $\begin{bmatrix} 8 & 0 \\ 11 & -9 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ 3 \end{bmatrix}$ 19. $\begin{bmatrix} 5 & 1 & -9 \\ 10 & 5 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$

21. $\begin{bmatrix} -7 & -11 \\ 36 & 8 \end{bmatrix} \begin{bmatrix} s \\ t \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ 23. $\begin{bmatrix} 4 & 22 & -12 \\ 33 & 3 & 3 \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 10 \\ -2 \end{bmatrix}$

25. $\begin{array}{r} \\ \text{Total sales} \\ \text{Total cost} \end{array}\begin{array}{c} \begin{array}{ccc} \text{Dec.} & \text{Apr.} & \text{Aug.} \end{array} \\ \begin{bmatrix} 11{,}240 & 6620 & 7840 \\ 7930 & 4680 & 5540 \end{bmatrix} \end{array}$

33. $\begin{bmatrix} -6 \\ -6 \\ 0 \end{bmatrix} = -2\begin{bmatrix} 9 \\ 0 \\ 3 \end{bmatrix} + 3\begin{bmatrix} 4 \\ -2 \\ 2 \end{bmatrix}$

35. $\begin{bmatrix} 8 \\ 21 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 8 \end{bmatrix} + (-1)\begin{bmatrix} 3 \\ -1 \end{bmatrix} + (-2)\begin{bmatrix} -4 \\ 2 \end{bmatrix}$

37. $\begin{bmatrix} -2 & 5 \\ 11 & -5 \end{bmatrix}$

39. $\begin{bmatrix} 7 & 6 & 5 \\ 19 & 2 & 5 \end{bmatrix}$

41. $\begin{bmatrix} 5 & -4 \\ 9 & 4 \end{bmatrix}$

SECTION 1.4

1. $e^{-1} = e, \begin{bmatrix} 0 & 0 & 1 & \\ 0 & 1 & 0 & \\ 1 & 0 & 0 & \\ & & & 1 \end{bmatrix}$

3. Multiply the third row by $\frac{3}{2}$, $\begin{bmatrix} 1 & & \\ & 1 & \\ & & \frac{2}{3} \end{bmatrix}, \begin{bmatrix} 1 & & \\ & 1 & \\ & & \frac{3}{2} \end{bmatrix}$

5. $E = \begin{bmatrix} 1 & & \\ & 0 & 1 \\ & 1 & 0 \end{bmatrix}, eA = EA = \begin{bmatrix} 1 & -2 & 3 & 0 \\ 3 & -1 & 1 & -1 \\ 4 & 1 & 1 & 7 \end{bmatrix}$

7. $E = \begin{bmatrix} 1 & 0 & -2 \\ & 1 & 0 \\ & & -1 \end{bmatrix}, eA = EA = \begin{bmatrix} -4 & -6 & 1 \\ 4 & -1 & 1 \\ 3 & 3 & 1 \end{bmatrix}$

9. Yes, add 2 times row 3 to row 1. 11. N 13. N

15. $E_1 = E_2 = \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 1 \end{bmatrix}$

17. Impossible because E_3E_1 is not an elementary matrix; that is, E_3E_1 requires *two* row exchanges to achieve this exchange of rows.

19. (a) E_1 *or* $E_2 = I$ (b) $E_1 = E_2 = P_{ij}$ (c) E_1 and E_2 multiply the same row by possibly different numbers. (d) E_1 and E_2 add (possibly different) multiples of the same first row to the same second row.

21. $DA = \begin{bmatrix} 15 & -10 & 30 \\ 26 & 8 & 20 \end{bmatrix}, BD = \begin{bmatrix} 30 & 2 \\ -60 & 30 \\ 35 & -4 \end{bmatrix}$

23. $DA = \begin{bmatrix} 0 & 0 & 0 \\ 4 & -4 & 6 \\ 5 & 3 & -12 \end{bmatrix}, BD = \begin{bmatrix} 0 & -16 & 2 \\ 0 & -4 & -3 \\ 0 & -6 & -12 \end{bmatrix}$

25. $\begin{bmatrix} 5 & -2 \\ 3 & -1 \end{bmatrix}$ 27. Does not exist 29. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ 0 & 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

31. $\begin{bmatrix} 1 & -2 & 1 & 0 \\ 1 & -2 & 2 & -3 \\ 0 & 1 & -1 & 1 \\ -2 & 3 & -2 & 3 \end{bmatrix}$ 33. $\begin{bmatrix} \frac{1}{3} & & \\ & 4 & \\ & & \frac{1}{5} \end{bmatrix}$

35. $\begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}, \begin{bmatrix} -2 & 5 \\ 1 & -2 \end{bmatrix}, \begin{bmatrix} -2 & 1 & & \\ 3 & -1 & & \\ & & -2 & 5 \\ & & 1 & -2 \end{bmatrix}$

37. (a) $\begin{bmatrix} a^{-1} & 0 \\ -a^{-1}b^{-1}c & b^{-1} \end{bmatrix}$ (b) $\begin{bmatrix} A^{-1} & \mathbf{0} \\ -B^{-1}CA^{-1} & B^{-1} \end{bmatrix}$

39. $C(B^{-1})^2A^{-1}$

41. $\begin{bmatrix} a^{-1} & & & \\ & b^{-1} & & \\ & & c^{-1} & \\ & & & d^{-1} \end{bmatrix}$ 43. $\begin{bmatrix} 0 & b^{-1} & & \\ a^{-1} & 0 & & \\ & & 0 & c^{-1} \\ & & d^{-1} & 0 \end{bmatrix}$

47. $\begin{bmatrix} 1 & \\ & 1 \end{bmatrix}, \begin{bmatrix} -1 & \\ & 1 \end{bmatrix}, \begin{bmatrix} 1 & \\ & -1 \end{bmatrix}, \begin{bmatrix} -1 & \\ & -1 \end{bmatrix}, \begin{bmatrix} & 1 \\ 1 & \end{bmatrix}$

49. Use 1.50(c): If $BX = \mathbf{0}$, then $ABX = A\mathbf{0} = \mathbf{0}$, so $X = \mathbf{0}$.

SECTION 1.5

1. $\begin{bmatrix} 1 & \\ 3 & 1 \end{bmatrix}\begin{bmatrix} 2 & 4 \\ & -9 \end{bmatrix}$ 3. $\begin{bmatrix} 1 & & \\ -1 & 1 & \\ 2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 3 \\ & 6 & 4 \\ & & -2 \end{bmatrix}$

5. $\begin{bmatrix} 1 & & \\ -2 & 1 & \\ -1 & \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 & -1 \\ & -8 & 0 \\ & & 3 \end{bmatrix}$

7. $\begin{bmatrix} 1 & & & \\ 2 & 1 & & \\ -3 & -11 & 1 & \\ 1 & 2 & -\frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 & 2 & -1 \\ & -1 & -1 & 4 \\ & & -6 & 43 \\ & & & \frac{31}{2} \end{bmatrix}$ 9. $\begin{bmatrix} -5 \\ -11 \end{bmatrix}$

11. $\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}$ 13. $\begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$ 15. $\begin{bmatrix} 2 \\ -1 \\ 3 \\ -2 \end{bmatrix}$ 17. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

19. $\begin{bmatrix} 2 \\ -3 \\ -1 \end{bmatrix}$ 21. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 23. $\begin{bmatrix} -\frac{1}{2} \\ -1 \\ 1 \end{bmatrix}$

25. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & \\ 0 & 1 \end{bmatrix}\begin{bmatrix} -3 & 4 \\ & 2 \end{bmatrix}$

27. $\begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 1 \end{bmatrix}\begin{bmatrix} 0 & 2 & 3 \\ -2 & 1 & 4 \\ 6 & -7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 0 & 1 & \\ -3 & -2 & 1 \end{bmatrix}\begin{bmatrix} -2 & 1 & 4 \\ & 2 & 3 \\ & & 20 \end{bmatrix}$

29. $\begin{bmatrix} 1 & & \\ & 0 & 1 \\ & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 3 & -1 \\ -4 & -6 & 5 \\ 4 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ -2 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 3 & -1 \\ & -5 & 4 \\ & & 3 \end{bmatrix}$

31. $\begin{bmatrix} 1 & & & \\ & 0 & 1 & 0 \\ & 0 & 0 & 1 \\ & 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 & 2 \\ 4 & -6 & 2 & 3 \\ 6 & -1 & -2 & 3 \\ -8 & -4 & -3 & -1 \end{bmatrix}$

$= \begin{bmatrix} 1 & & & \\ -3 & 1 & & \\ 4 & -2 & 1 & \\ -2 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -2 & 3 & -1 & 2 \\ & 8 & -5 & 9 \\ & & -9 & 9 \\ & & & 7 \end{bmatrix}$

33. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ 35. $\begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$ 39. **HINT** If $i > j$, $u_{ij} = 0$.

SECTION 1.6

1. $\begin{bmatrix} 2 & 4 \\ -3 & 1 \end{bmatrix}$ 3. $\begin{bmatrix} 5 & 8 & -4 \\ 7 & -1 & 2 \\ 2 & -3 & 4 \end{bmatrix}$ 5. $\begin{bmatrix} 2 & -1 & 3 & 0 \\ 4 & 7 & -3 & -2 \\ 9 & 1 & 3 & -1 \end{bmatrix}$

7. $\begin{bmatrix} 2 & -5 \\ -1 & 3 \end{bmatrix}, \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 5 \\ 1 & 2 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 2 & -3 & 4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 & 3 & -4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}, \begin{bmatrix} 1 & & & \\ -2 & 1 & & \\ 3 & & 1 & \\ -4 & & & 1 \end{bmatrix},$

$\begin{bmatrix} 1 & -2 & 3 & -4 \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix}$

11. $\begin{bmatrix} 1 & & \\ 3 & 1 & \\ 2 & & 1 \end{bmatrix}, \begin{bmatrix} 1 & & \\ -3 & 1 & \\ -2 & & 1 \end{bmatrix}, \begin{bmatrix} 1 & -3 & -2 \\ & 1 & \\ & & 1 \end{bmatrix}, \begin{bmatrix} 1 & & \\ -3 & 1 & \\ -2 & & 1 \end{bmatrix}$

13. $A^T = D(C^T)^2B^T$, $A^{-1} = (D^{-1})^T(C^{-1})^2B^{-1}$, $(A^T)^{-1} = (B^{-1})^T[(C^{-1})^T]^2D^{-1}$

15. (a) $\begin{bmatrix} 1 & & \\ 1 & 1 & \\ & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 & \\ & 1 & 1 \\ & & 2 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \\ -\frac{3}{2} & \frac{3}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

17. (a) $\begin{bmatrix} 1 & & & \\ 2 & 1 & & \\ & 4 & 1 & \\ & & -2 & 1 \end{bmatrix}\begin{bmatrix} 2 & \frac{1}{2} & & \\ & 1 & 1 & \\ & & -2 & 1 \\ & & & 4 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} & 0 & -\frac{1}{16} & \frac{1}{32} \\ 0 & 0 & \frac{1}{4} & -\frac{1}{8} \\ -2 & 1 & -\frac{1}{4} & \frac{1}{8} \\ 4 & -2 & \frac{1}{2} & \frac{1}{4} \end{bmatrix}$ (c) $\begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}$

23. $\begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}$, (itself); $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, (itself); $\begin{bmatrix} & & 1 \\ & 1 & \\ 1 & & \end{bmatrix}$, (itself);
$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, (itself); $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \leftrightarrow \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

SECTION 1.7

1. (a) (0, 2.000) (b) (1.000, 2.000) 3. (2, −3) 5. (3, −2, 0)

7. $(\frac{1}{2}, -2, 3)$ 9. (−2, 3, −1, 4)

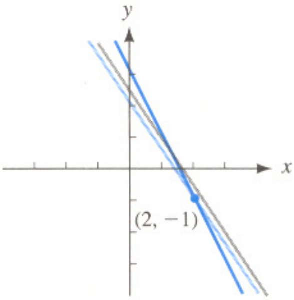
11. $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 1 & \\ \frac{1}{2} & 1 \end{bmatrix}\begin{bmatrix} 4 & -3 \\ & \frac{5}{2} \end{bmatrix}$

13. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} -2 & 3 & 1 \\ 4 & -8 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ \frac{1}{4} & 1 & \\ -\frac{1}{2} & -\frac{1}{3} & 1 \end{bmatrix}\begin{bmatrix} 4 & -8 & 3 \\ & 3 & \frac{1}{4} \\ & & \frac{31}{12} \end{bmatrix}$

15. $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 4 & -2 & 1 \\ -2 & 3 & -1 \\ 6 & 12 & 6 \end{bmatrix} = \begin{bmatrix} 1 & & \\ \frac{2}{3} & 1 & \\ -\frac{1}{3} & -\frac{7}{10} & 1 \end{bmatrix}\begin{bmatrix} 6 & 12 & 6 \\ & -10 & -3 \\ & & -\frac{11}{10} \end{bmatrix}$

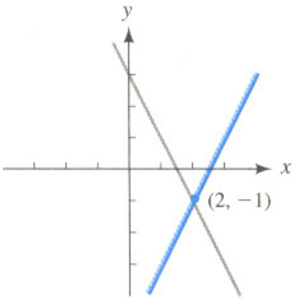
17. $$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & -2 & 3 & 1 \\ 2 & 1 & 1 & -4 \\ 4 & 8 & 4 & -1 \\ 1 & 1 & 1 & 1 \end{bmatrix} =$$
$$\begin{bmatrix} 1 & & & \\ \frac{3}{4} & 1 & & \\ \frac{1}{2} & \frac{3}{8} & 1 & \\ \frac{1}{4} & \frac{1}{8} & 0 & 1 \end{bmatrix}\begin{bmatrix} 4 & 8 & 4 & -1 \\ & -8 & 0 & \frac{7}{4} \\ & & -1 & -\frac{133}{32} \\ & & & \frac{33}{32} \end{bmatrix}$$

19.



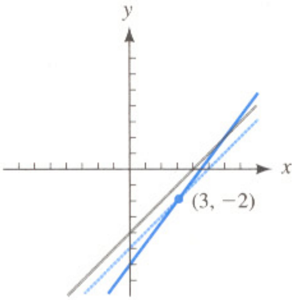
Ill-conditioned,
new solution: (1, 1)

21.



Not ill-conditioned,
new solution: (2.001, −1.002)

23.



Ill-conditioned,
new solution: (6, 2)

CHAPTER 1 REVIEW

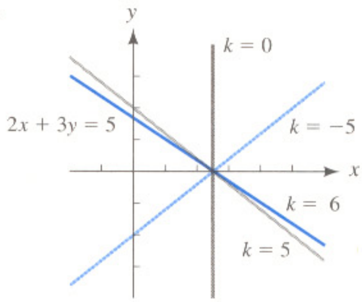
1. (a) $\begin{bmatrix} 2 & 3 & -1 \\ 4 & -1 & -2 \\ -2 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 4 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{2} + \frac{1}{2}t \\ -2 \\ t \end{bmatrix}$

3. (a) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 4 \\ -3 \end{bmatrix}$ (b) $\begin{bmatrix} 2 \\ -1 \\ 2 \\ 2 \end{bmatrix}$

5. $\begin{bmatrix} -5 & 2 \\ -3 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 \\ -3 & 2 \end{bmatrix}$ 7. $\begin{bmatrix} 1 & -2 \\ & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -2 \\ 3 & -5 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ 11. (a) A with 20 times row 1 added to row 2 (b) $\begin{bmatrix} 1 & \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & \\ 20 & 1 \end{bmatrix}, \begin{bmatrix} 10 & \\ 20 & 10 \end{bmatrix}$

13.



(a) None, $k \neq 6$, $k = 6$
(b) k near 6, $k \neq 6$

15. (a) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (c) $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

17. (a), (b) a, b, c all nonzero 19. $\begin{bmatrix} -\frac{7}{2} & 2 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} 1 & \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 4 \\ & -1 \end{bmatrix}$

21. Column 1 of A = Column 1 of B. *That's all!*

23. $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 2 \\ 0 & 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 0 & 1 & \\ 0 & \frac{2}{3} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 2 \\ & 3 & 4 \\ & & \frac{1}{3} \end{bmatrix}; \begin{bmatrix} 2 \\ -1 \\ -2 \end{bmatrix}$

25. (a) If the single 1 in column 1 of P is in the ith row, let P_1 interchange rows 1 and i. Next, look at column 2 of P_1P, and so on. (b) $P = P_1 \cdots P_{n-1}$

29. $[A^{-1} + B^{-1}]^{-1} = B(A + B)^{-1}A$ 31. (a) T (b) F (c) F (d) T

SECTION 2.1

1. 0 3. 0 5. −8 7. 0 9. −1 11. 0 13. 6
15. 100 17. −1284 19. 420 23. 3 25. 30 27. $\det[a] = a$

SECTION 2.2

1. −1, −13, 13 3. 2, −20, −40 5. 1, −5, −5

7. 30, −72, −2160   9. N   11. N   13. Y   17. −24, −24
19. 3, −1, −3   21. 6, −2, −12   23. 13, 1, 13   25. n is even.

SECTION 2.3

1. 3, odd   3. 5, odd   5. 3, odd   7. Y, +   9. N   11. Y, −
13. 0   15. −5   17. 21   19. 27   21. −6   23. $x + 2$
25. $x^3 + 2x^2 - 3x$ 27. −12   29. 1   31. 0
33. Each elementary product has 0 as a factor.   35. $n(n - 1)/2$

SECTION 2.4

1. $M_{11} = 4, M_{12} = 3, M_{21} = 1, M_{22} = -2$
$C_{11} = 4, C_{12} = -3, C_{21} = -1, C_{22} = -2$
3. $M_{11} = -2, M_{12} = 0, M_{13} = 1, M_{21} = 0, M_{22} = 0, M_{23} = 2$
$M_{31} = 2, M_{32} = 4, M_{33} = -3$
$C_{11} = -2, C_{12} = 0, C_{13} = 1, C_{21} = 0, C_{22} = 0, C_{23} = -2$
$C_{31} = 2, C_{32} = -4, C_{33} = -3$
5. −11   7. −4   9. −20   11. 28   13. 27   15. 6
17. 1   19. −48   21. $(b - a)(c - a)(c - b)$

23. $\begin{bmatrix} 1 & -4 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}, \frac{1}{6}\begin{bmatrix} 1 & 1 \\ -4 & 2 \end{bmatrix}$

25. $\begin{bmatrix} 0 & 2 & -1 \\ -1 & 11 & -7 \\ -1 & 8 & -5 \end{bmatrix}, \begin{bmatrix} 0 & -1 & -1 \\ 2 & 11 & 8 \\ -1 & -7 & -5 \end{bmatrix}, \frac{1}{1}\begin{bmatrix} 0 & -1 & -1 \\ 2 & 11 & 8 \\ -1 & -7 & -5 \end{bmatrix}$

27. $\begin{bmatrix} -8 & 4 & 0 \\ -1 & -2 & 0 \\ -26 & 8 & 20 \end{bmatrix}, \begin{bmatrix} -8 & -1 & -26 \\ 4 & -2 & 8 \\ 0 & 0 & 20 \end{bmatrix}, -\frac{1}{20}\begin{bmatrix} -8 & -1 & -26 \\ 4 & -2 & 8 \\ 0 & 0 & 20 \end{bmatrix}$

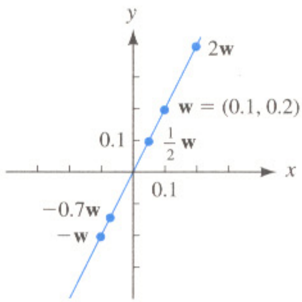
29. $(3, -2)$ 31. $(-2, \frac{1}{2}, 3)$ 33. $(3, -2, 0)$

CHAPTER 2 REVIEW

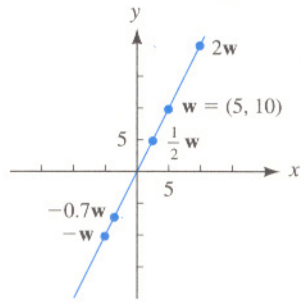
1. −1   3. −1
5. Yes. Use the definition, and a sum of products of integers is an integer.
9. $[\det(A + B)]^2 = \det(A + B)^2 = \det[A^2 + AB + BA + B^2] = \det[A^2 + B^2]$
11. −12   13. No, $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

SECTION 3.1

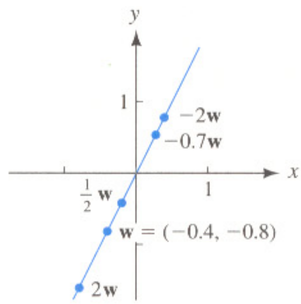
1. $(-1, 1)$ 3. $(-5, 11, -2)$ 5. $(-1, 4, 6)$ 7. $(-1, 0, -3)$
9. $(-8, 20, 40)$
11.



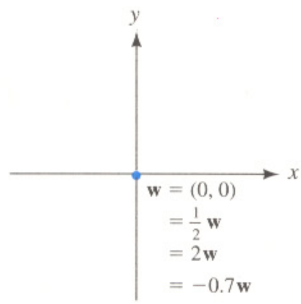
(*a*)



(*b*)

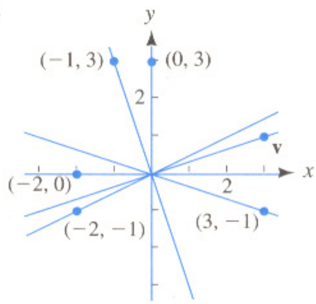


(*c*)

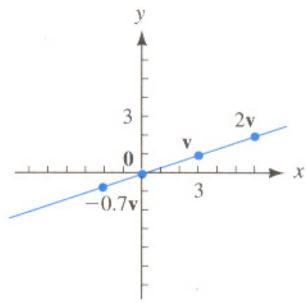


(*d*)

13.



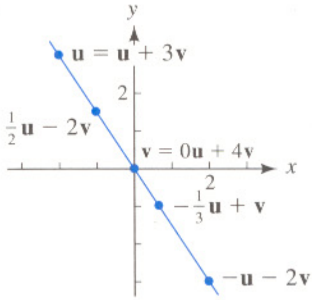
(*a*)



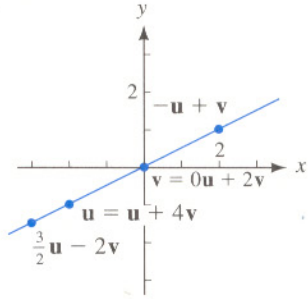
(*b*)

(*c*) When one is a multiple of the other.

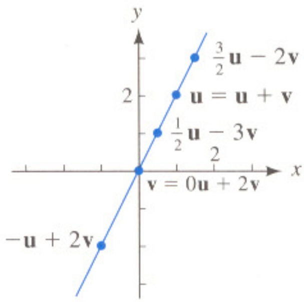
15.



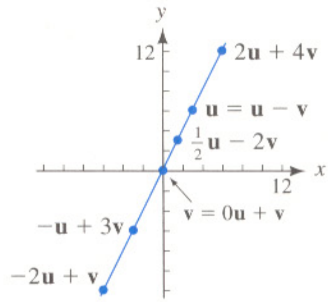
(*a*)



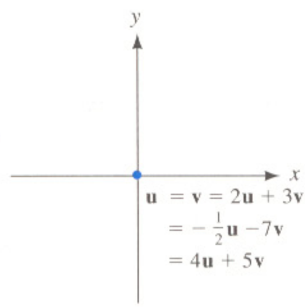
(*b*)



(*c*)

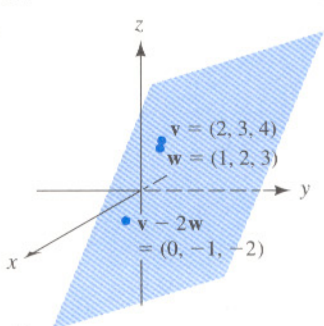


(*d*)

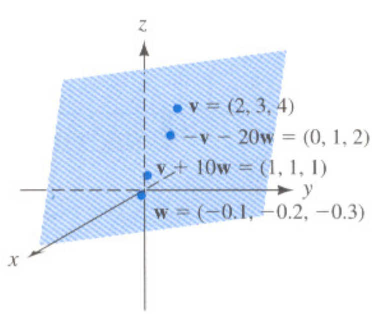


(*e*)

17.

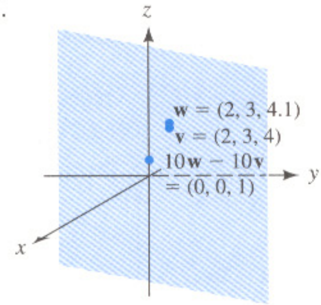


(*a*)

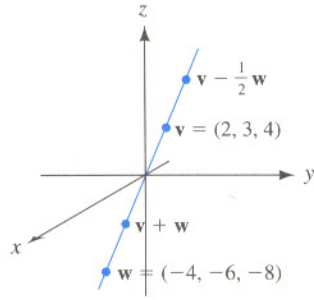


(*b*)

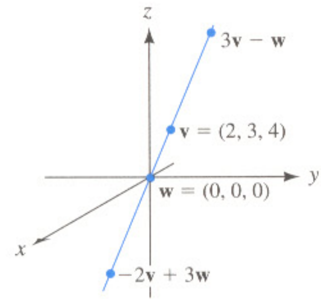
17.



(*c*)



(*d*)



(*e*)

19. (a) When $\mathbf{w}$ is a multiple of $\mathbf{v}$. The line through $\mathbf{v}$ and the origin.
(b) The origin; a line (through the origin); a plane (through the origin).

21. (a) A plane (b) A plane (c) A line (d) A line

23. $2\sqrt{2}$ 25. $\sqrt{73}$ 27. 5 29. $6\sqrt{14}$

31. 9, 0° 33. 0, 90° 35. 7, 69.4859°

37. (a) $\mathbf{w} = \mathbf{0}$ (b) $\mathbf{p} = \mathbf{0}$

39. (a) $\mathbf{w} = \frac{13}{10}\mathbf{v} = \left(-\frac{13}{5}, \frac{26}{5}\right)$ (b) $\mathbf{p} = \left(\frac{28}{5}, \frac{14}{5}\right)$

41. (a) $\mathbf{w} = \frac{11}{13}\mathbf{v} = \left(\frac{33}{13}, \frac{44}{13}, -\frac{11}{13}\right)$ (b) $\mathbf{p} = \left(\frac{71}{13}, -\frac{57}{13}, -\frac{15}{13}\right)$

43. $\mathbf{p}_3 = (0, 0, 7)$ 45. $\mathbf{p}_3 = \left(\frac{2}{3}, \frac{1}{3}, -\frac{1}{3}\right)$ 47. $\mathbf{p}_3 = \left(\frac{1}{10}, -\frac{7}{10}, 1\right)$

49. $\mathbf{p}_1 = (1, 0, 0)$, $\mathbf{p}_2 = (0, 1, 0)$, $\mathbf{p}_3 = (0, 0, 1)$

SECTION 3.2

1. $(-2, 6, -8, 6, -5)$ 3. $(-9, -8, -7, -5, 3, 9)$

5. $(5, -3, 16, 3, 5)$ 7. $\begin{bmatrix} -1 \\ 0 \\ -3 \\ -4 \end{bmatrix}$ 9. $\begin{bmatrix} -8 \\ 20 \\ 40 \\ 28 \end{bmatrix}$ 11. $\sqrt{21}$

13. $\sqrt{22}$ 15. 3 17. $\sqrt{493}$ 19. 1, $\|r\mathbf{w}\| = |r|\,\|\mathbf{w}\|$
21. 31, 0° 23. 0, 90° 25. 2, 83.9576° 27. $\mathbf{0}$
29. $\frac{5}{11}\mathbf{v} = \begin{bmatrix} \frac{20}{11} & \frac{10}{11} & \frac{5}{11} & 0 & \frac{5}{11} \end{bmatrix}$ 31. $\frac{12}{19}\mathbf{v} = \left(\frac{36}{19}, \frac{24}{19}, -\frac{12}{19}, \frac{12}{19}, \frac{24}{19}\right)$

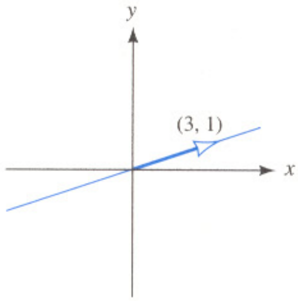
SECTION 3.3

1. (b) is $((r+s)x_1, 2x_2)$
13. Note that f is continuous where it is defined, but $f(0)$ is not defined.
17. N, Axiom (h) 19. N, Axiom (f)
21. Y 23. N, closure of scalar multiplication 25. Y
27. N, Axiom (c) 29. Y 37. $\int_0^{2\pi} \sin x \cos x\, dx = 0, \mathbf{0}$ 39. π

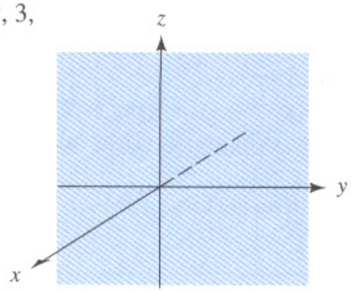
SECTION 3.4

1. Y, Y, Y 3. Y, N, N 5. Y, Y, Y 7. Y, Y, Y
9. Y, Y, Y 11. Y, Y, Y 13. Y, Y, Y 15. Y, Y, Y
17. Y, Y, Y 19. N, Y, N 21. Y, Y, Y 23. Y, Y, Y

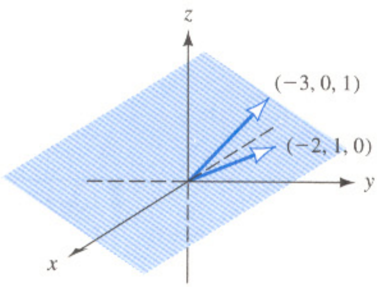
25. $\left\{ x\begin{bmatrix} 3 \\ 1 \end{bmatrix} \middle| \, x \text{ in } \mathbb{R} \right\}$, 2,



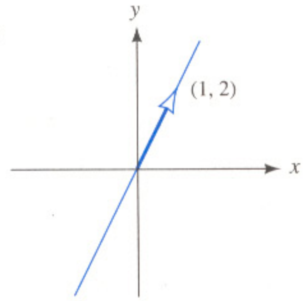
27. $\mathbb{R}^3$, 3,



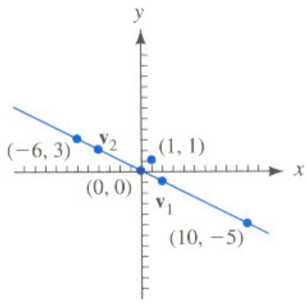
29. $\left\{ y\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + z\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \middle| \, y, z \text{ in } \mathbb{R} \right\}$, 3,



31. $\left\{ x \begin{bmatrix} 1 \\ 2 \end{bmatrix} \middle| \, x \text{ in } \mathbb{R} \right\}$, 2,

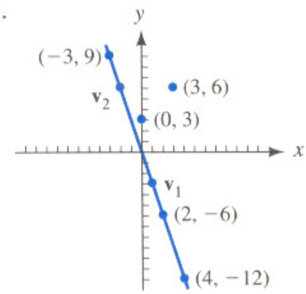


33.



(a) Y, −3, 0 (b) N
(c) Y, 0, 0 (d) Y, 5, 0

35.



(a) Y, −1, 1 (b) N
(c) Y, 0, −1
(d) N (e) Y, −2, −3

37. (a) N (b) Y, 0, 0 (c) Y, 3, −2 (d) Y, 2, −5
39. (a) Y, −1, 1 (b) N (c) Y, 0, 0 (d) Y, 1, 1 (e) N
41. The main diagonal in $\mathbb{R}^2$ 43. The xy-plane in $\mathbb{R}^3$
45. All diagonal 2×2 matrices 47. All polynomials of degree ≤ 1

SECTION 3.5

1. $\mathbf{u}_2 = 3\mathbf{u}_1$ 3. $\mathbf{w}_3 = 0\mathbf{w}_1 + 0\mathbf{w}_2$ 5. $A_3 = 2A_1 - A_2$
7. Line, dep. 9. Line, dep. 11. Larger, indep.
13. Dep., Theorem (3.46) 15. Indep., by inspection
17. Dep., by inspection 19. Dep., $p_3 = -2p_1 + 3p_2$ 21. Indep.
23. Indep. 25. Dep.
31. Suppose that $c_1\mathbf{x}_1 + \cdots + c_k\mathbf{x}_k = \mathbf{0}$. Then $c_1A\mathbf{x}_1 + \cdots + c_kA\mathbf{x}_k = A\mathbf{0}$ or $c_1\mathbf{b}_1 + \cdots + c_k\mathbf{b}_k = \mathbf{0}$. Thus all $c_i = 0$ since the $\mathbf{b}$'s are linearly independent.
33. Definitely not; five vectors in $\mathbb{R}^3$

SECTION 3.6

1. $3 > \dim \mathbb{R}^2$ 3. $3 < \dim \mathbb{R}^4$ 5. $2 < \dim P_2$ 7. Y

9. N 11. Y 13. N 15. Y 17. Y 19. N

21. $\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, etc.; 6 23. $\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$, etc.; mn 25. N

27. $\left\{\begin{bmatrix} 2 \\ -\frac{3}{2} \\ 1 \end{bmatrix}\right\}$, 1 29. $\left\{\begin{bmatrix} -\frac{4}{5} \\ \frac{6}{5} \\ 1 \end{bmatrix}\right\}$, 1 31. No basis, 0

33. $\left\{\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right\}$, 1 35. $\left\{\begin{bmatrix} 1 & & \\ & 0 & \\ & & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & \\ 1 & 0 & \\ & & 0 \end{bmatrix}, \text{etc.}\right\}$, 6

37. Infinite dimensional

39. (b) {(1, −1, 0, 0), (1, 0, −1, 0), (1, 0, 0, −1)}, 3

41. (a) $\begin{bmatrix} 1 & 1 & 0 & -1 \\ 1 & -1 & -1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 1 \end{bmatrix}$, 2

43. Take the first two; add $\boldsymbol{\varepsilon}_3 = [0 \quad 0 \quad 1]$.

45. Take the first, second, and fourth; add $\boldsymbol{\varepsilon}_3 = [0 \quad 0 \quad 1 \quad 0]$.

47. (a) {(1, 1, 0), (0, 0, 1)} (b) Add $\boldsymbol{\varepsilon}_1 = (1, 0, 0)$ (c) N

49. (a) T (b) F 51. Must be [see Theorem (3.64c)]

SECTION 3.7

1. [2 4 3], [0 −2 4], etc., $\begin{bmatrix} 2 \\ 0 \\ 3 \\ -1 \end{bmatrix}$, etc.

3. [1 −2 5 2 1 3], [0 0 2 3 0 −1], [0 0 0 0 4]; $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}$; $rk = 3$

5. Every row; every column; $rk = 4$

7. [1 2 −3], [0 0 11]; $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ 5 \\ 0 \end{bmatrix}$; $rk = 2$

9. [0 2 −3 1 2], [0 0 0 4 3]; $\begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \\ 6 \end{bmatrix}$; $rk = 2$

11. (2, −1, 6), (0, 5, −14) 13. $\begin{bmatrix} 2 \\ 3 \\ 2 \\ -4 \end{bmatrix}, \begin{bmatrix} 0 \\ -5 \\ -8 \\ 10 \end{bmatrix}$

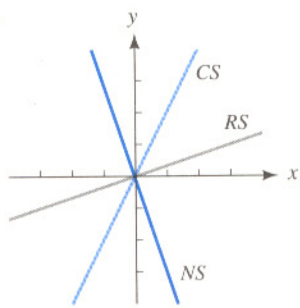
15. $2 - 3x + 4x^2 - 5x^3,\ 5x + 7x^2 + 8x^3$

17. $\{[2 \quad -1 \quad 3], [0 \quad 4 \quad -5]\}$; $\left\{\begin{bmatrix} -\frac{7}{8} \\ \frac{5}{4} \\ 1 \end{bmatrix}\right\}$; 2; 1

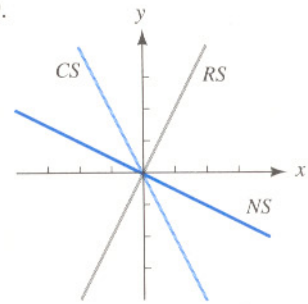
19. All 3 rows; no basis; 3; 0 21. $\mathbf{b} = 5\mathbf{c}_1 + 1\mathbf{c}_2$ 23. N

25. $\mathbb{R}^3$, $\mathbb{R}^3$, $\{\mathbf{0}\}$

27.



29.



31. *yz*-plane, *xy*-plane, *x*-axis

39. $[1 \quad 3 \quad -2 \quad 4], [0 \quad 0 \quad 5 \quad 1]$; $\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 22 \\ 0 \\ 1 \\ -5 \end{bmatrix}$; $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix}$; $[0 \quad 0 \quad 1]$

43. (a) (b) F, $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ (c) T, $\begin{bmatrix} 1 \\ 0 \end{bmatrix}\mathbf{x} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ has no solution.

(d) T, four vectors in $\mathbb{R}^3$ (e) F, $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

SECTION 3.8

1. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ 3. $\begin{bmatrix} -\frac{9}{4} \\ -\frac{13}{4} \end{bmatrix}, \begin{bmatrix} -\frac{13}{4} \\ -\frac{9}{4} \end{bmatrix}$ 5. $\begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$

7. $\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 3 \end{bmatrix}$ 9. $\begin{bmatrix} 2 \\ \frac{4}{3} \\ -\frac{1}{6} \end{bmatrix}, \begin{bmatrix} -\frac{1}{6} \\ \frac{4}{3} \\ 2 \end{bmatrix}$ 11. $\begin{bmatrix} 2 \\ -1 \\ 3 \\ -6 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 2 \\ -6 \end{bmatrix}$

13. $\mathbf{x} = \frac{2}{5}\mathbf{v}_1 - \frac{1}{5}\mathbf{v}_2 + 0\mathbf{v}_3 = -\frac{4}{5}\mathbf{v}_1 + \frac{7}{5}\mathbf{v}_2 + 2\mathbf{v}_3 = -\frac{1}{5}\mathbf{v}_1 + \frac{3}{5}\mathbf{v}_2 + \mathbf{v}_3$

$\mathbf{y} = \frac{4}{5}\mathbf{v}_1 - \frac{7}{5}\mathbf{v}_2 - \mathbf{v}_3 = \frac{1}{5}\mathbf{v}_1 - \frac{3}{5}\mathbf{v}_2 - 0\mathbf{v}_3 = -\frac{2}{5}\mathbf{v}_1 + \frac{1}{5}\mathbf{v}_2 + \mathbf{v}_3$

15. $[a_1 \cdots a_n]^T$, $[b_1 \cdots b_n]^T$, $[a_1 + b_1 \cdots a_n + b_n]^T$

17. $\begin{bmatrix} \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} \end{bmatrix}, \begin{bmatrix} -\frac{3}{4} \\ \frac{5}{8} \end{bmatrix}$ 19. $\begin{bmatrix} 2 & -1 \\ 3 & -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

21. $\begin{bmatrix} -7 & -6 & 12 \\ 6 & 5 & -10 \\ -4 & -4 & 7 \end{bmatrix}, \begin{bmatrix} 7 \\ -6 \\ 3 \end{bmatrix}$ 23. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$

25. $\begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 3 \\ 0 \end{bmatrix}$ 27. $\begin{bmatrix} \frac{1}{4} & -\frac{1}{4} \\ \frac{1}{8} & \frac{3}{8} \end{bmatrix}, \begin{bmatrix} 3 & 2 \\ -1 & 2 \end{bmatrix}$

29. $\begin{bmatrix} -7 & -6 & 12 \\ 6 & 5 & -10 \\ -4 & -4 & 7 \end{bmatrix}, \begin{bmatrix} 5 & 6 & 0 \\ 2 & 1 & -2 \\ 4 & 4 & -1 \end{bmatrix}$ 31. QP

SECTION 3.9

1. (0, 1, 1, 0, 0, 1, 1) 3. (0, 1, 1, 1, 1, 0, 0) 5. Y; (0, 1, 1, 0)
7. N; (0, 1, 1, 0) 9. N; (0, 0, 0, 1) 11. N; (1, 1, 0, 0)
13. (0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1); 8
15. 2; 2 17. 4; 4 19. 7(3) = 21; $(2^n - 1)(2^n - 2)/2$

CHAPTER 3 REVIEW

1. $\begin{bmatrix} 5 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -5 \\ 5 \\ -15 \\ 14 \end{bmatrix}$ 3. $\frac{1}{\sqrt{10}}\begin{bmatrix} 0 \\ 3 \\ 1 \\ 0 \end{bmatrix}$, many others

5. {(1, 0, 0, 0), (0, 1, 1, 1)} 7. $\begin{bmatrix} 2 & 1 & -1 & 0 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$; 2; 2; 3

9. N

11. Y; $A_3 = 2A_1 - A_2$ 13. $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} + t\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$; none

15. **HINT** A^{-1} exists. 17. (a) F (b) T (c) F (d) T (e) F

19. (b), (d), (e) 21. (c), (d)

23. (a) $\{[1 \quad 0 \quad 0]^T, [2 \quad 3 \quad 0]^T\} = \{\mathbf{c}_2, \mathbf{c}_3\}$

(b) $\mathbf{c}_1 = 0\mathbf{c}_2 + 0\mathbf{c}_3$, $\mathbf{c}_2 = 1\mathbf{c}_2 + 0\mathbf{c}_3$, $\mathbf{c}_3 = 0\mathbf{c}_2 + 1\mathbf{c}_3$, $\mathbf{c}_4 = 2c_2 + 1\mathbf{c}_3$

(c) $\begin{bmatrix} 0 & 1 & 2 & 4 \\ 0 & 0 & 3 & 3 \\ 0 & 1 & 2 & 4 \end{bmatrix}$ 25. $\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right\}$

27. (a), (b) $m = n = r$ (c) $r = n < m$ (d) $m = r < n$

SECTION 4.1

1. $f_A(x_1, x_2) = (x_1 + 2x_2, -2x_1, 3x_1 - x_2)$; $f_A : \mathbb{R}^2 \to \mathbb{R}^3$
3. $f_A(x_1, x_2) = (4x_1 - x_2, x_1 + 2x_2, 3x_1, -x_1 + x_2)$; $f_A : \mathbb{R}^2 \to \mathbb{R}^4$
5. $f_A(x_1, x_2, x_3, x_4) = x_1 + 2x_2 - x_3 + 3x_4$; $f_A : \mathbb{R}^4 \to \mathbb{R}^1$
7. Expands the x-axis (multiplies it by 2), contracts the y-axis $\left(\text{multiplies it by } \frac{1}{2}\right)$
9. Interchanges the x- and y-axes
11. Rotates both axes by 45° (see Example 5)

13. $\begin{bmatrix} \frac{1}{2}\sqrt{3} & -\frac{1}{2} & \\ \frac{1}{2} & \frac{1}{2}\sqrt{3} & \\ & & 1 \end{bmatrix}$ 15. $\begin{bmatrix} -1 & & \\ & \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ & \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$

17. Y; $T : \mathbb{R}^2 \to \mathbb{R}^2$; $\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ 19. N; $T : \mathbb{R}^2 \to \mathbb{R}^2$

21. Y; $T : \mathbb{R}^3 \to \mathbb{R}^2$; $\begin{bmatrix} 2 & 1 & 0 \\ 1 & -1 & 1 \end{bmatrix}$ 23. Y; $T : \mathbb{R}^3 \to \mathbb{R}^3$; $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

25. Y; $T : \mathbb{R}^2 \to \mathbb{R}^3$; $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & -3 \end{bmatrix}$ 27. Y 29. N 31. Y

33. Y 35. Y 37. Y 39. N 41. (a) Y (b) $J : C[a, b] \to \mathbb{R}$

SECTION 4.2

1. 1, 1; 7, 7 3. 5, 5, 5; −10, −10, −10
5. $\sqrt{13} \approx 3.6$, $\sqrt{6} + \sqrt{3} \approx 4.2$; $\sqrt{33} \approx 5.7$, $\sqrt{10} + \sqrt{13} \approx 6.8$

7. $-1, [-1], \begin{bmatrix} 2 & 1 & -1 \\ -2 & -1 & 1 \\ 4 & 2 & -2 \end{bmatrix}$, (a) = (b);

$2, \begin{bmatrix} 2 & -2 & -2 & 2 \\ -1 & 1 & 1 & -1 \\ 2 & -2 & -2 & 2 \\ 1 & -1 & -1 & 1 \end{bmatrix}$, [2], (a) = (c).

9. Y 11. N, (a) 13. Y 15. N, (d) 17. -14, $[-14]$

19. 3, [3] 21. (a) 19, 19 (b) -49, -49 23. (a) -82, -82 (b) 20, 20

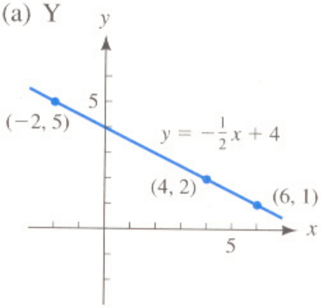
25. (a) $\begin{bmatrix} 38 & -76 \\ -76 & 152 \end{bmatrix}$ (b), (c) $\left\{ x\begin{bmatrix} 2 \\ 1 \end{bmatrix} \middle| x \text{ in } \mathbb{R} \right\}$ (d), (e) 1

27. (a) $\begin{bmatrix} 15 & -30 & 0 \\ -30 & 60 & 0 \\ 0 & 0 & 74 \end{bmatrix}$ (b), (c) $\left\{ x\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \middle| x \text{ in } \mathbb{R} \right\}$ (d), (e) 2

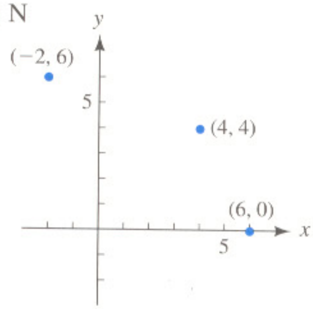
41. Only for $(a, a, \ldots, a)$

SECTION 4.3

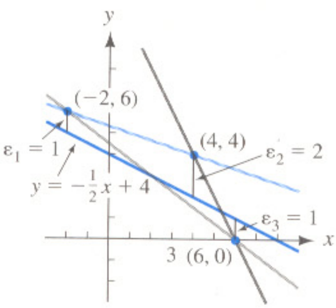
1. (a) Y



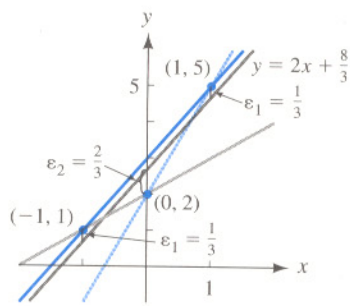
(b) N



3. $6\frac{1}{4}, \frac{100}{9}, 100, 6; \left(-\frac{1}{2}, 4\right)$



5. $1, 4, 4, \frac{2}{3}; \left(2, \frac{8}{3}\right)$



7. $y = \frac{1}{4}x + \frac{1}{6}$ 9. $y = \frac{2}{5}x - \frac{4}{5}$ 11. $y = 2x + \frac{8}{3}$ 13. $y = 1.3x + 1.8$

15. $y = \frac{19}{14}x$ 17. $p = -\frac{4}{3}\begin{bmatrix}1\\1\\1\end{bmatrix}, P = \frac{1}{3}\begin{bmatrix}1&1&1\\1&1&1\\1&1&1\end{bmatrix}$

19. $p = \begin{bmatrix}-1\\0\\-1\end{bmatrix}, P = \frac{1}{3}\begin{bmatrix}2&-1&1\\-1&2&1\\1&1&2\end{bmatrix}$

21. $p = (1, 2, 3, 4, 0, 0)$, $P = \begin{bmatrix}1&&&&&\\&1&&&&\\&&1&&&\\&&&1&&\\&&&&0&\\&&&&&0\end{bmatrix}$

23. $\begin{bmatrix}4&2&6\\2&6&8\\6&8&18\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}6\\5\\11\end{bmatrix}$ 25. $\begin{bmatrix}4&2&2\\2&2&1\\2&1&2\end{bmatrix}\begin{bmatrix}a\\b\\c\end{bmatrix} = \begin{bmatrix}9\\3\\3\end{bmatrix}$

27. $\begin{bmatrix}6&6&3&3\\6&10&3&5\\3&3&3&3\\3&5&3&5\end{bmatrix}\begin{bmatrix}a\\b\\c\\d\end{bmatrix} = \begin{bmatrix}20\\27\\11\\14\end{bmatrix}$ 29. $\begin{bmatrix}4&6\\6&14\end{bmatrix}\begin{bmatrix}\ln a\\\ln b\end{bmatrix} = \begin{bmatrix}33\\74\end{bmatrix}$

SECTION 4.4

1. Y 3. N 7. (a) 6, −4, 1 (b) $\frac{5}{3}, 5\sqrt{2}, -\frac{1}{3}\sqrt{2}$

9. (a) 1, 2, 3, 4, 5, 6 (b) 6, 5, 4, 3, 2, 1 11. $y = \frac{3}{2}x + \frac{2}{3}$; $\begin{bmatrix}-1&1\\0&1\\1&1\end{bmatrix}$

13. $y = -1.5x - 3.8$; $\begin{bmatrix}-2&1\\-1&1\\0&1\\1&1\\2&1\end{bmatrix}$ 15. $y = 0.4x + 0.5$; $\begin{bmatrix}-\frac{3}{2}&1\\-\frac{1}{2}&1\\\frac{1}{2}&1\\\frac{3}{2}&1\end{bmatrix}$

17. $(1/\sqrt{2}, 1/\sqrt{2}), (-1/\sqrt{2}, 1/\sqrt{2})$ 19. $\begin{bmatrix}1/\sqrt{3}\\1/\sqrt{3}\\1/\sqrt{3}\end{bmatrix}, \begin{bmatrix}-2/\sqrt{6}\\1/\sqrt{6}\\1/\sqrt{6}\end{bmatrix}, \begin{bmatrix}0\\-1/\sqrt{2}\\1/\sqrt{2}\end{bmatrix}$

21. $\begin{bmatrix} \frac{2}{3} \\ 0 \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}, \begin{bmatrix} \frac{1}{3} \\ 0 \\ -\frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}, \begin{bmatrix} 2/3\sqrt{10} \\ 3/\sqrt{10} \\ -1/3\sqrt{10} \\ 2/3\sqrt{10} \end{bmatrix}$ 23. $\frac{1}{\sqrt{2}}, \sqrt{\frac{3}{2}}x, \frac{1}{4}\sqrt{10}(3x^2-1)$

25. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}, \begin{bmatrix} -2/\sqrt{6} & 1/\sqrt{6} \\ 1/\sqrt{6} & 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{30} & 4/\sqrt{30} \\ -2/\sqrt{30} & -3/\sqrt{30} \end{bmatrix}$

SECTION 4.5

1. $\begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}$ 3. $\begin{bmatrix} \frac{4}{5} \\ -\frac{3}{5} \\ 0 \end{bmatrix}$ 5. $\begin{bmatrix} \frac{2}{7} & -\frac{6}{7} & \frac{3}{7} \end{bmatrix}$ 7. Y, $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

9. N 11. Y, $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/\sqrt{2} & 0 & -1/\sqrt{2} \\ 0 & 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 0 & 1/\sqrt{6} & -2/\sqrt{6} & 1/\sqrt{6} \end{bmatrix}$

13. $\begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{3}{5} & -\frac{4}{5} \end{bmatrix}\begin{bmatrix} 5 & \frac{1}{5} \\ & \frac{7}{5} \end{bmatrix}$ 15. $\begin{bmatrix} \frac{1}{3} & 8/3\sqrt{26} \\ \frac{2}{3} & 7/3\sqrt{26} \\ -\frac{2}{3} & 11/3\sqrt{26} \end{bmatrix}\begin{bmatrix} 3 & \frac{1}{3} \\ & \sqrt{26}/3 \end{bmatrix}$

17. $y = \frac{3}{2}x - 2$ 19. $y = -\frac{61}{114}x + \frac{35}{114}$

SECTION 4.6

1. (a) The plane spanned by the x-axis and the line $z = y$ in the yz-plane
(b) (1, 3, 2), (4, 1, 1), (0, 2, −2)

3. (a) $Q\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -x_2 \\ -x_1 \end{bmatrix}$ (b) $-\boldsymbol{\varepsilon}_2, -\boldsymbol{\varepsilon}_1 - \boldsymbol{\varepsilon}_2, \boldsymbol{\varepsilon}_1 - \boldsymbol{\varepsilon}_2$

5. (a) $Q\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 \\ \frac{3}{5}x_2 + \frac{4}{5}x_3 \\ \frac{4}{5}x_2 - \frac{3}{5}x_3 \end{bmatrix}$ (b) $\boldsymbol{\varepsilon}_1, \begin{bmatrix} 1 \\ \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}, \begin{bmatrix} 1 \\ -\frac{3}{5} \\ -\frac{4}{5} \end{bmatrix}$

7. (a) $Q\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{3}x_1 + \frac{1}{3}x_2 + \frac{2}{3}x_3 \\ \frac{1}{3}x_1 + \frac{2}{3}x_2 - \frac{2}{3}x_3 \\ \frac{2}{3}x_1 - \frac{2}{3}x_2 - \frac{1}{3}x_3 \end{bmatrix}, \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$ (b) $\boldsymbol{\varepsilon}_1 + \boldsymbol{\varepsilon}_2, \begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3} \\ \frac{4}{3} \end{bmatrix}$

9. (a) $Q\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}x_1 - \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{1}{2}x_4 \\ -\frac{1}{2}x_1 + \frac{1}{2}x_2 - \frac{1}{2}x_3 + \frac{1}{2}x_4 \\ -\frac{1}{2}x_1 - \frac{1}{2}x_2 + \frac{1}{2}x_3 + \frac{1}{2}x_4 \\ \frac{1}{2}x_1 + \frac{1}{2}x_2 + \frac{1}{2}x_3 + \frac{1}{2}x_4 \end{bmatrix}, \begin{bmatrix} \frac{1}{2} \\ -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

(b) $-\boldsymbol{\varepsilon}_3 + \boldsymbol{\varepsilon}_4, \boldsymbol{\varepsilon}_1 - \boldsymbol{\varepsilon}_2$

11. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

13. $\begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\sqrt{2} \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} & 0 \end{bmatrix}$

15. $\begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$

17. $\begin{bmatrix} 2/\sqrt{5} \\ -1/\sqrt{5} \end{bmatrix}, \begin{bmatrix} \frac{1}{5} \\ \frac{7}{5} \end{bmatrix}$

19. $\begin{bmatrix} 2/\sqrt{6} \\ 1/\sqrt{6} \\ -1/\sqrt{6} \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

21. $\begin{bmatrix} -3/2\sqrt{3} \\ -1/2\sqrt{3} \\ 1/2\sqrt{3} \\ -1/2\sqrt{3} \end{bmatrix}, \begin{bmatrix} -1 \\ \frac{1}{3} \\ \frac{2}{3} \\ -\frac{2}{3} \end{bmatrix}$

23. (a) $\begin{bmatrix} -2 & -1 \\ 1/\sqrt{2} & 3 \\ 1/\sqrt{2} & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} -2 & -1 \\ & -3 \end{bmatrix}$

25. (a) $\begin{bmatrix} 3 & -\frac{5}{3} \\ -2/\sqrt{6} & 3.350 \\ 1/\sqrt{6} & 1.026 \\ -1/\sqrt{6} & 0.050 \end{bmatrix}$

27. (a) $\begin{bmatrix} \sqrt{2} & -\sqrt{2}/2 & -\sqrt{2}/2 \\ \sqrt{2}/2 & 1.225 & 0.408 \\ \frac{1}{2} & -0.996 & -1.155 \\ \frac{1}{2} & 0.0848 & -1 \end{bmatrix}$

29. (a) $\begin{bmatrix} -2 & 2 \\ 3/2\sqrt{3} & -\sqrt{2} \\ 1/2\sqrt{3} & (1+\sqrt{2})/\sqrt{4+2\sqrt{2}} \\ 1/2\sqrt{3} & -1/\sqrt{4+2\sqrt{2}} \\ 1/2\sqrt{3} & 0 \end{bmatrix}$

31. (a) $\begin{bmatrix} -2 & -2 & -1 \\ 3/\sqrt{12} & 1 & -1 \\ \sqrt{2}/\sqrt{12} & 1/\sqrt{2} & \sqrt{3} \\ 1/\sqrt{12} & 0 & (-1-\sqrt{3})/\sqrt{6+2\sqrt{3}} \\ 0 & 1/\sqrt{2} & \sqrt{2}/\sqrt{6+2\sqrt{3}} \end{bmatrix}$

SECTION 4.7

1. (a) $\begin{bmatrix} 1 & 1 \\ 1 & -1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} -\frac{3}{2} & 1 \\ \frac{1}{2} & -1 \\ -\frac{1}{2} & 0 \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} -2 \\ 6 \\ 2 \end{bmatrix}$

3. (a) $\begin{bmatrix} -1 & 0 & 1 \\ -1 & 2 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3} & -1 & -\frac{2}{3} \\ \frac{1}{3} & 1 & \frac{1}{3} \end{bmatrix}$ (c) $\begin{bmatrix} -2 \\ -3 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} -5 \\ -4 \end{bmatrix}$

5. (a) $\begin{bmatrix} 1 & -1 \\ 3 & 0 \\ 0 & -2 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 2 \\ 4 & 4 \\ -7 & -6 \\ 7 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} -3 \\ 2 \end{bmatrix}$ (d) $\begin{bmatrix} 7 \\ 3 \\ 12 \\ -5 \end{bmatrix}$

7. (a) $\begin{bmatrix} 1 & -1 \\ 1 & 1 \\ 2 & 0 \\ 0 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 3 & -3 \\ -1 & 5 \\ 0 & -2 \\ -2 & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 6 & 4 \\ 10 & -3 \end{bmatrix}$

9. (a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 0 & -2 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ -\frac{5}{2} \end{bmatrix}$ (d) $3x^2 - 2x^3$

11. (a) $\begin{bmatrix} 2 \\ 3 \\ -1 \\ -2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 3 \\ 0 \\ -3 \\ -2 \end{bmatrix}$

13. See 1(b), $f_A(x, y) = (2x + y, -x, -3y)$

15. See 3(b), $f_A(x, y, z) = (-x + z, x + y)$

17. See 5(b), $f_A(x, y) = (3x, -x + y, 2y, -y)$

23. $\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & -1 \\ 2 & 2 \end{bmatrix}$ 25. $\begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}, \begin{bmatrix} -1 & 1 \\ 2 & -1 \end{bmatrix}$

27. $\begin{bmatrix} 1 & 0 & -1 \\ 1 & -1 & 1 \\ 0 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & -1 \\ 1 & -2 & -1 \\ 0 & 2 & 2 \end{bmatrix}$

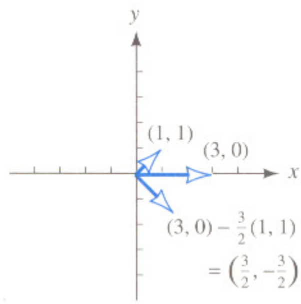
29. $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 2 & 2 \\ 0 & 1 & -1 \\ -1 & -1 & 0 \end{bmatrix}$

31. Use $P = I$ 35. Same P 41. Use $P = B$

CHAPTER 4 REVIEW

1. Y, $\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ 1 & 4 \end{bmatrix}$ 7. (a) $\frac{2}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ (b) $\frac{1}{14}\begin{bmatrix} -8 \\ 5 \\ 31 \end{bmatrix}$

9. $\frac{3}{2}$,



11. $(0, 0, 1/\sqrt{2}, 1/\sqrt{2})$ 13. $\begin{bmatrix} \frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}\begin{bmatrix} 5 & \frac{6}{5} \\ & \frac{8}{5} \end{bmatrix}$

15. (a) $\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ (b) Interchange $x_1 \leftrightarrow x_3$, $x_2 \leftrightarrow x_4$

17. 3; (0, 1, 1), (4, 1, −1) 19. Y; A, B orthogonal $\Rightarrow$ AB orthogonal

21. 16×16 23. [1 2 −3]; does not exist

25. $\mathbf{0} = 0\mathbf{u}_1 + 0\mathbf{u}_2$ 27. $\begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

29. **HINT** First show that $CS(AA^T) \subset CS(A)$. Then use dimensions.

31. $\|A\mathbf{x} - \mathbf{b}\|$ or $\|A\mathbf{x} - \mathbf{b}\|^2$

33. (a) **HINT** You can solve $A\mathbf{x} = \mathbf{b}$; can you also solve $AA^T\mathbf{y} = \mathbf{b}$? See Exercise 29.

(b) **HINT** If $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x}' = \mathbf{b}$, where is $\mathbf{x} - \mathbf{x}'$? Now use Exercise 30.

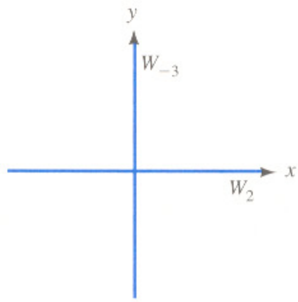
SECTION 5.1

1. (a) 3.1 (b) Y 3. (a) 0 (b) N 5. (a) 7 (b) Y
7. (a) 0 (b) N 9. (a) 4 (b) Y 11. (a) 25 (b) Y
13. (a) 0 (b) N 15. (a) −8 (b) Y 17. (a) −12 (b) Y
19. (a) 0 (b) N 21. (a) x (b) 0 23. (a) $x^2 - 2x - 3$ (b) 3, −1
25. (a) $x + 3$ (b) −3 27. (a) $x^2 + x - 12$ (b) 3, −4

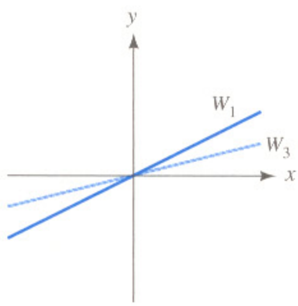
29. (a) $24x - 13$ (b) $\frac{13}{24}$ 31. (a) $(x - 2)(x + 3)(x + 1)(x - 5)$
(b) $-3, -1, 2, 5$ 33. ab 35. $125a$ 37. c

SECTION 5.2

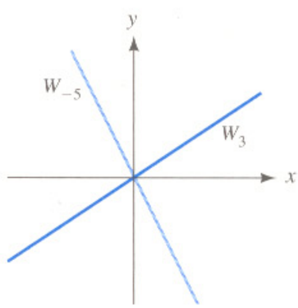
1. 6, 4 3. 2, 0 5. 1, . . . , 1 7. $A\mathbf{v} = \mathbf{0} = 0\mathbf{v}$; 0

11. (a) $(\lambda - 2)(\lambda + 3)$ (b) 2, -3 (c) $\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2$
(d)


(f) $2 + (-3) = 2 + (-3)$

13. (a) $(\lambda - 1)(\lambda - 3)$ (b) 1, 3 (c) $[2 \quad 1]^T$, $[4 \quad 1]^T$
(d)


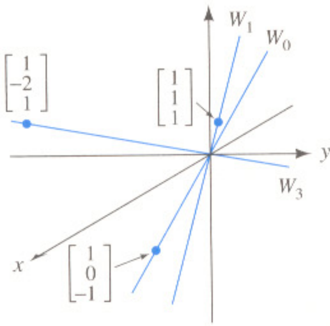
(f) $5 + (-1) = 1 + 3$

15. (a) $(\lambda - 3)(\lambda + 5)$ (b) 3, -5 (c) $[3 \quad 2]^T$, $[-1 \quad 2]^T$
(d)


(f) $1 + (-3) = 3 + (-5)$

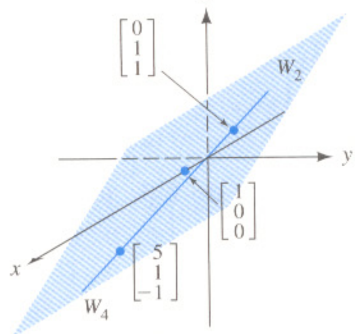
17. (a) $\lambda(\lambda - 1)(\lambda - 3)$ (b) 0, 1, 3 (c) $[1 \quad 1 \quad 1]^T$, $[1 \quad 0 \quad -1]^T$, $[1 \quad -2 \quad 1]^T$

(d)



(f) $1 + 2 + 1 = 0 + 1 + 3$

19. (a) $(\lambda - 2)^2(\lambda - 4)$ (b) 2, 2, 4 (c) $[1 \quad 0 \quad 0]^T$, $[0 \quad 1 \quad 1]^T$, $[5 \quad -1 \quad 1]^T$
(d)



(f) $2 + 3 + 3 = 2 + 2 + 4$

21. (a) $(\lambda + 2)(\lambda - 3)^2(\lambda + 6)$ (b) $-2, 3, 3, -6$
(c) $[2 \quad 1 \quad 0 \quad 0]^T$, $[1 \quad 3 \quad 0 \quad 0]^T$, $[0 \quad 0 \quad 1 \quad -2]^T$, $[0 \quad 0 \quad 4 \quad 1]^T$
(f) $-3 + 4 + (-5) + 2 = -2 + 3 + 3 + (-6)$

23. λ^2 25. λ^{-1} 27. $\lambda - 7$

31. (a) $\{\mathbf{u}_1\}$, $\{\mathbf{u}_2, \mathbf{u}_3\}$ (b) $\mathbf{x} = \frac{1}{2}\mathbf{u}_2 + \frac{1}{4}\mathbf{u}_3$
(c) $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are linearly independent, and $\mathbf{u}_1$ is not in $CS(A)$, by (a).

35. (a), (b) $A = \begin{bmatrix}2 & 1\\0 & 1\end{bmatrix}$, $B = \begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$
(d) $\det(AB) = \det(A)\det(B)$, $\mathrm{tr}(A + B) = \mathrm{tr}(A) + \mathrm{tr}(B)$

SECTION 5.3

1. (a) $\begin{bmatrix}1 & 1\\1 & 0\end{bmatrix}$, $\begin{bmatrix}2 & \\ & -3\end{bmatrix}$ (b) $\begin{bmatrix}-3 & 5\\0 & 2\end{bmatrix}$

3. (a) $\begin{bmatrix}1 & 0 & 1\\0 & 1 & 0\\1 & 0 & -1\end{bmatrix}$, $\begin{bmatrix}-2 & & \\ & -1 & \\ & & 0\end{bmatrix}$ (b) $\begin{bmatrix}-1 & 0 & -1\\0 & -1 & 0\\-1 & 0 & -1\end{bmatrix}$

5. (a) $\begin{bmatrix}1&&&&\\&1&&&\\&&1&&\\&&&1&\\&&&&1\end{bmatrix}, \begin{bmatrix}2&&&&\\&3&&&\\&&4&&\\&&&5&\\&&&&6\end{bmatrix}$ (b) $A = \Lambda$

13. $\begin{bmatrix}1&1\\-1&2\end{bmatrix}, \begin{bmatrix}3&\\&2\end{bmatrix}$ 15. $\begin{bmatrix}1&1&1\\1&0&1\\0&0&1\end{bmatrix}, \begin{bmatrix}3&&\\&0&\\&&-2\end{bmatrix}$

17. N 19. $\begin{bmatrix}4&1\\1&-1\end{bmatrix}, \begin{bmatrix}2&\\&-3\end{bmatrix}$ 21. $\begin{bmatrix}1&0&0\\0&1&2\\0&0&1\end{bmatrix}, \begin{bmatrix}1&&\\&1&\\&&2\end{bmatrix}$

23. N 25. $\begin{bmatrix}1&0&0&0\\0&1&1&1\\0&0&-1&0\\0&0&0&1\end{bmatrix}, \begin{bmatrix}3&&&\\&3&&\\&&-2&\\&&&-2\end{bmatrix}$

29. $\frac{1}{3}\begin{bmatrix}62&-19\\-38&43\end{bmatrix}, \frac{1}{18}\begin{bmatrix}7&1\\2&8\end{bmatrix}$

31. Show the columns of S are eigenvectors of A.

33. Yes; the eigenvalues are all distinct; $\begin{bmatrix}1&&\\&3&\\&&-6\end{bmatrix}$

35. From the standard basis to the basis of eigenvectors.

37. $f_A(\mathbf{v}_i) = \lambda_i\mathbf{v}_i$, so $\mathbf{v}_i$ is moved farther from the origin if $\lambda_i > 1$; left fixed if $\lambda_i = 1$; moved closer to the origin if $0 < \lambda_i < 1$; collapsed to $\mathbf{0}$ if $\lambda_i = 0$; and reflected through the origin if $\lambda_i < 0$.

SECTION 5.4

1. (a) $\begin{bmatrix}1\\2\end{bmatrix}, \begin{bmatrix}2\\-1\end{bmatrix}$ (b) $\begin{bmatrix}1/\sqrt{5}&2/\sqrt{5}\\2/\sqrt{5}&-1/\sqrt{5}\end{bmatrix}, \begin{bmatrix}5&\\&0\end{bmatrix}$

3. (a) $\begin{bmatrix}2\\-1\end{bmatrix}, \begin{bmatrix}1\\2\end{bmatrix}$ (b) $\begin{bmatrix}2/\sqrt{5}&1/\sqrt{5}\\-1/\sqrt{5}&2/\sqrt{5}\end{bmatrix}, \begin{bmatrix}6&\\&1\end{bmatrix}$

5. (a) $\begin{bmatrix}1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\end{bmatrix}$ (b) $\begin{bmatrix}1/\sqrt{2}&1/\sqrt{2}\\1/\sqrt{2}&-1/\sqrt{2}\end{bmatrix}, \begin{bmatrix}2&\\&0\end{bmatrix}$

7. (a) $\begin{bmatrix}1\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}1\\1\\-2\end{bmatrix}$ (b) $\begin{bmatrix}1/\sqrt{3}&1/\sqrt{2}&1/\sqrt{6}\\1/\sqrt{3}&-1/\sqrt{2}&1/\sqrt{6}\\1/\sqrt{3}&0&-2/\sqrt{6}\end{bmatrix}, \begin{bmatrix}3&&\\&0&\\&&0\end{bmatrix}$

9. (a) $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -5 \end{bmatrix}$ (b) $\begin{bmatrix} \frac{1}{3} & 2/\sqrt{5} & 2/3\sqrt{5} \\ -\frac{2}{3} & 1/\sqrt{5} & -4/3\sqrt{5} \\ \frac{2}{3} & 0 & -5/3\sqrt{5} \end{bmatrix}, \begin{bmatrix} 6 & & \\ & -3 & \\ & & -3 \end{bmatrix}$

11. (a) $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ -1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 4 & & \\ & -1 & \\ & & 1 \end{bmatrix}$

13. (a) $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ -3 \end{bmatrix}$

(b) $\begin{bmatrix} \frac{1}{2} & 1/\sqrt{2} & 1/\sqrt{6} & 1/2\sqrt{3} \\ \frac{1}{2} & -1/\sqrt{2} & 1/\sqrt{6} & 1/2\sqrt{3} \\ \frac{1}{2} & 0 & -2/\sqrt{6} & 1/2\sqrt{3} \\ \frac{1}{2} & 0 & 0 & -3/2\sqrt{3} \end{bmatrix}, \begin{bmatrix} 4 & & & \\ & 0 & & \\ & & 0 & \\ & & & 0 \end{bmatrix}$

15. $\begin{bmatrix} 2 & \\ & -3 \end{bmatrix}, \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} \\ 1/\sqrt{5} & -2/\sqrt{5} \end{bmatrix}$

17. $\begin{bmatrix} 3 & & \\ & -4 & \\ & & -1 \end{bmatrix}, \begin{bmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & \frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$

19. $\begin{bmatrix} 2 & & \\ & 3 & \\ & & 3 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & -1/\sqrt{2} & 1/\sqrt{6} \\ 1/\sqrt{3} & 0 & -1/\sqrt{6} \end{bmatrix}$

SECTION 5.5

1. 1, 1, 2, 3, 5, 8

3. 1, 3, 4, 7, 11, 18

9. $S_{k+1} = \frac{1}{2}F_k$, $T_{k+1} = \frac{1}{3}S_k$, $F_{k+1} = 6T_k$

13. $\begin{bmatrix} E_{k+1} \\ F_{k+1} \\ S_{k+1} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 100 \\ \frac{1}{20} & 0 & 0 \\ 0 & \frac{1}{5} & 0 \end{bmatrix} \begin{bmatrix} E_k \\ F_k \\ S_k \end{bmatrix}$

21. Half in, half out; only the fixed population

23. $\begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

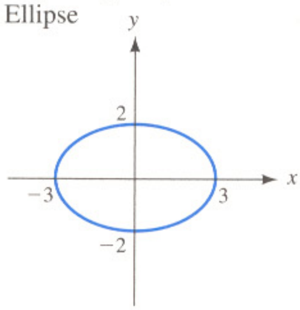
25. (a) $\begin{bmatrix} 64 \\ 64 \\ 64 \end{bmatrix}$ (b) $\begin{bmatrix} 32 \\ 32 \\ 128 \end{bmatrix} \to \begin{bmatrix} 80 \\ 80 \\ 32 \end{bmatrix} \to \begin{bmatrix} 56 \\ 56 \\ 80 \end{bmatrix} \to \begin{bmatrix} 68 \\ 68 \\ 56 \end{bmatrix} \to \begin{bmatrix} 62 \\ 62 \\ 68 \end{bmatrix} \to \begin{bmatrix} 65 \\ 65 \\ 62 \end{bmatrix}$

SECTION 5.6

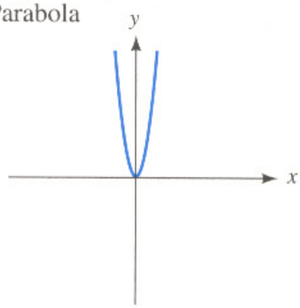
1. (a) $\begin{bmatrix} u \\ v \end{bmatrix} = ae^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + be^{-2t}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$ (b) $a = -11,\ b = 6$
3. (a) $\begin{bmatrix} u \\ v \end{bmatrix} = ae^{t}\begin{bmatrix} 2 \\ 1 \end{bmatrix} + be^{3t}\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ (b) $a = \frac{5}{2},\ b = -\frac{1}{2}$
5. (a) $\begin{bmatrix} u \\ v \end{bmatrix} = ae^{-t}\begin{bmatrix} 1 \\ -1 \end{bmatrix} + be^{7t}\begin{bmatrix} 3 \\ 1 \end{bmatrix}$ (b) $a = -2,\ b = 3$
7. (a) $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = a\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + be^{t}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + ce^{3t}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$ (b) $a = 4,\ b = -1,\ c = 0$
9. (a) $\begin{bmatrix} u \\ v \\ w \end{bmatrix} = ae^{-6t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + be^{2t}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + ce^{4t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ (b) $a = 2,\ b = 3,\ c = -1$
11. (a) $y = ae^{2t} + be^{-t}$ (b) $a = 3,\ b = 2$
13. (a) $y = ae^{t} + be^{2t} + ce^{3t}$ (b) $a = \frac{13}{2},\ b = -9,\ c = \frac{7}{2}$

SECTION 5.7

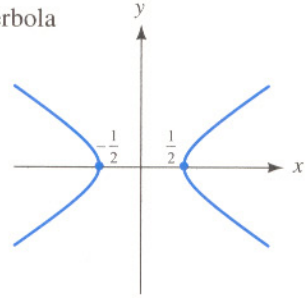
1. $2x^2 + 4xy + y^2$
3. $3x^2 + xy - 2y^2 - 8yz + 2z^2 - 5xz$
5. Ellipse



7. Parabola



9. Hyperbola



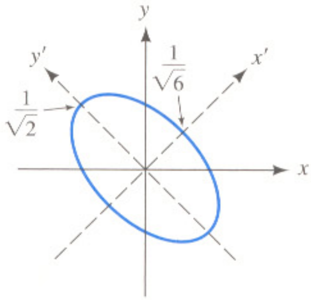
11. $[x \quad y]\begin{bmatrix} 3 & 2 \\ 2 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

13. $[x \quad y]\begin{bmatrix} 5 & -1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

15. $[x \quad y \quad z]\begin{bmatrix} 2 & -4 & \frac{1}{2} \\ -4 & -5 & -3 \\ \frac{1}{2} & -3 & -7 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

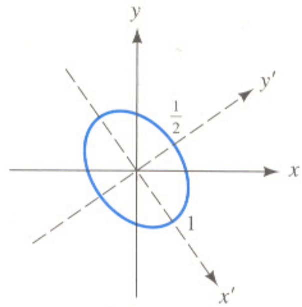
17. $[x \quad y \quad z]\begin{bmatrix} 2 & 0 & -\frac{1}{2} \\ 0 & 9 & \frac{3}{2} \\ -\frac{1}{2} & \frac{3}{2} & 0 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

19. $6x'^2 + 2y'^2$



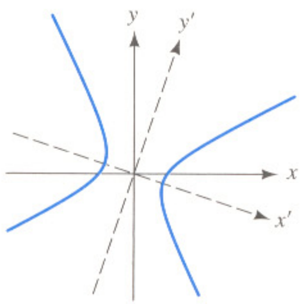
$\mathbf{x}' = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}, \mathbf{y}' = \begin{bmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

21. $x'^2 + 4y'^2$



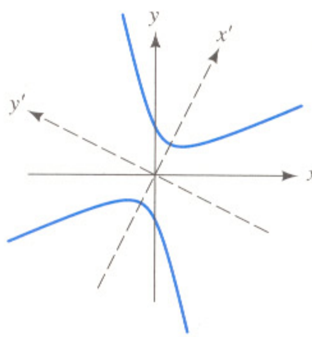
$\mathbf{x}' = \begin{bmatrix} 1/\sqrt{3} \\ -\sqrt{2}/\sqrt{3} \end{bmatrix}, \mathbf{y}' = \begin{bmatrix} \sqrt{2}/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$

23. $\frac{5}{2}x'^2 - \frac{5}{2}y'^2$



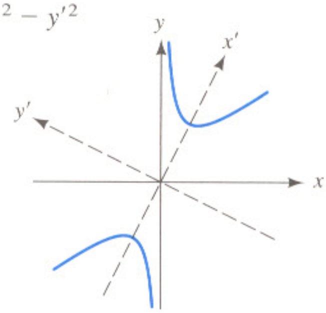
$\mathbf{x}' = \begin{bmatrix} 3/\sqrt{10} \\ -1/\sqrt{10} \end{bmatrix}, \mathbf{y}' = \begin{bmatrix} 1/\sqrt{10} \\ 3/\sqrt{10} \end{bmatrix}$

25. $2x'^2 - 3y'^2$



$\mathbf{x}' = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}, \mathbf{y}' = \begin{bmatrix} -2\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

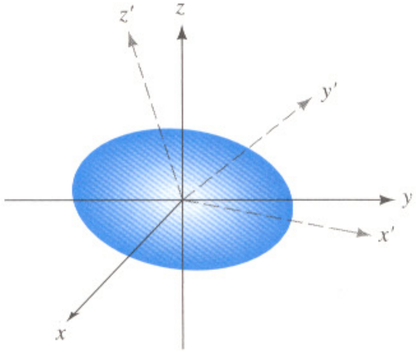
27. $\frac{1}{4}x'^2 - y'^2$



$\mathbf{x}' = \begin{bmatrix} 1/\sqrt{5} \\ 2/\sqrt{5} \end{bmatrix}$

$\mathbf{y}' = \begin{bmatrix} -2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$

29. $6x'^2 + 9y'^2 + 12z'^2 = 1$



$$\mathbf{x}' = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, \mathbf{y}' = \begin{bmatrix} -\frac{2}{3} \\ \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}, \mathbf{z}' = \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \\ \frac{2}{3} \end{bmatrix}$$

SECTION 5.8

1. $B\mathbf{w} = (S^{-1}AS)(S^{-1}\mathbf{v}) = S^{-1}A\mathbf{v} = S^{-1}\lambda\mathbf{v} = \lambda\mathbf{w}$

3. 3, $\begin{bmatrix} 4/\sqrt{17} \\ 1/\sqrt{17} \end{bmatrix}$; 1, $\begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$ 5. -5, $\begin{bmatrix} 1/\sqrt{5} \\ -2/\sqrt{5} \end{bmatrix}$; 3, $\begin{bmatrix} 3/\sqrt{13} \\ 2/\sqrt{13} \end{bmatrix}$

7. 6, $\begin{bmatrix} 4/\sqrt{17} \\ 1/\sqrt{17} \end{bmatrix}$; 4, $\begin{bmatrix} 2/\sqrt{5} \\ 1/\sqrt{5} \end{bmatrix}$ 9. 4.5, 1.4 11. 1.7, 3.3 13. 0.5, 0.5

15. 10, 0.6; 1, 2.1 17. 5, 0.6; -2, 1

CHAPTER 5 REVIEW

1. 1 3. $x^2 - 4x + 11$ 5. $\begin{bmatrix} 0 & 1 \\ 3 & -2 \end{bmatrix}$

7. $S = \begin{bmatrix} 2 & 2 \\ 3 & -1 \end{bmatrix}$, $\Lambda = \begin{bmatrix} 7 & \\ & 1 \end{bmatrix}$

11. (a) The eigenvectors are orthogonal. (b) 0 (c) 1, $\frac{1}{10}\begin{bmatrix} 9 & -3 \\ -3 & 1 \end{bmatrix}$

13. F; identity matrix 15. (a) ± 1 (b) 0, -1 (c) $\begin{bmatrix} 3 & -1 \\ 8 & -3 \end{bmatrix}$

17. (a) T (b), (c) May not; $\begin{bmatrix} 2 & 1 & \\ & 2 & \\ & & -3 \end{bmatrix}$ 19. $F_{-k} = (-1)^{k+1}F_k$

21. $\begin{bmatrix} u \\ v \end{bmatrix} = 2e^{2t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} - e^{-3t}\begin{bmatrix} 1 \\ -4 \end{bmatrix}$

23. $S = \begin{bmatrix} 1/\sqrt{5} & 1/\sqrt{5} \\ -2/\sqrt{5} & 2/\sqrt{5} \end{bmatrix}, \Lambda = \begin{bmatrix} 1 & \\ & 5 \end{bmatrix}$

SECTION 6.1

5. f is g_0 of Equation (6.4) 7. $b_n = \dfrac{2}{n}, n \geq 1$

9. $f(x) = \pi - 2\sin x - \sin 2x - \frac{2}{3}\sin 3x - \cdots$
$= \pi - 2\sum_{n=1}^{\infty} \dfrac{1}{n}\sin nx, 0 \leq x \leq 2\pi$

11. $f(x) = \frac{4}{3}\pi^2 + 4\sum_{n=1}^{\infty}\left(\dfrac{1}{n^2}\cos nx - \dfrac{\pi}{n}\sin nx\right), 0 \leq x \leq 2\pi$

13. $b_n = \dfrac{4}{n\pi}$, n odd; $b_n = 0$, n even; $f(x) = \dfrac{4}{\pi}\{\sin x + \frac{1}{3}\sin 3x + \cdots\}$

15. $q_3(x) = x^3 - \frac{3}{5}x$, $q_4(x) = x^4 - \frac{6}{7}x^2 + \frac{3}{35}$ 17., 19. $y = x - \frac{1}{6}$

21., 23. $y = \frac{9}{10}x - \frac{1}{5}$

SECTION 6.2

1. 2, 1 3. 2 5. $\sqrt{5}$ 7. $\sqrt{(9 \pm \sqrt{17})/2}$

9. $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 3 & \\ & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 11. $[1][\sqrt{5} \quad 0]\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}^T$

13. $\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}\begin{bmatrix} 5 & \\ & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} \\ -2/\sqrt{5} & 1/\sqrt{5} \end{bmatrix}^T$

15. $\begin{bmatrix} \sqrt{2}/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & -\sqrt{2}/\sqrt{3} \end{bmatrix}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 1/\sqrt{3} & \sqrt{2}/\sqrt{3} & 0 \\ \sqrt{2}/\sqrt{3} & -1/\sqrt{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$

17. $\begin{bmatrix} 1/\sqrt{5} & 2/\sqrt{5} & 0 \\ 2/\sqrt{5} & -1/\sqrt{5} & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{bmatrix} 3 & & \\ & 2 & \\ & & 2 \end{bmatrix}\begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{5} & 0 \\ 1/\sqrt{5} & -2/\sqrt{5} & 0 \\ 0 & 0 & 1 \end{bmatrix}^T$

19. $\begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ & \cdots & & \\ 1 & & & \end{bmatrix}\begin{bmatrix} n & & & & \\ & n-1 & & & \\ & & n-2 & & \\ & & & \ddots & \\ & & & & 1 \end{bmatrix}\begin{bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ & \cdots & & \\ 1 & & & \end{bmatrix}^T$