1. §14, # 12: The problem is to compute the order of the element $x := (3,1)+\langle(1,1)\rangle$ in the factor group $(\mathbf{Z}_4 \times \mathbf{Z}_4)/\langle(1,1)\rangle$. Thus, x is the coset of the cyclic group $H := \langle(1,1)\rangle$ containing $(3,1)$. It is helpful to note that $H = \{(0,0),(1,1),(2,2),(3,3)\}$. Thus the order of x is the smallest positive integer m such that $(3,1)$ belongs to H. Note that $(3,1)$ does not belong to H. On the other hand, $2(3,1) = (6,2) = (2,2) \in H$. Thus x has order 2.

2. §14 # 34. Let G be a group and H a subgroup. Suppose that H is the only subgroup of G of some given order m. Then H is normal. Indeed, to prove that H is normal, it suffices to show that for every g in G, the set $gHg^{-1} := \{ghg^{-1} : h \in H\}$ is H. But this set gHg^{-1} is another subgroup of G, and has the same order as H, and hence must be equal to H.