Problem 0.18 Let A be a set, let $B := \{0, 1\}$, and let B^A denote the set of functions from A to B. The problem is to exhibit a "natural" bijection between B^A and the power set $\mathcal{P}(A)$ of A. To do this, let S be an element of $\mathcal{P}(A)$, that is, a subset of A, and define a function $\chi_S \colon A \to B$ by the rule: $\chi_S(a) = 0$ if $a \notin S$ and $\chi_S(A) = 1$ if $a \in S$. This is clearly a well-defined function (mapping) from A to B. Hence the rule $S \mapsto \chi_S$ is a well-defined function from $\mathcal{P}(A)$ to B^A. It is perhaps almost obvious by now that knowledge of the subset S is completely equivalent to knowledge of χ_S, but let us write it our carefully. Suppose we start with an element of B^A, that is, a function $f \colon A \to B$. Define $\sigma(f)$ to be the set of all elements a of A such that that $f(a) = 1$. Then $\sigma(f) \in \mathcal{P}(A)$, and $f \mapsto \sigma(f)$ defines a mapping $B^A \to \mathcal{P}(A)$. Now it is immediate to check that for S, $\sigma(\chi_S) = S$ and for any f, $\chi_{\sigma(f)} = f$. This shows that $S \mapsto \chi_S$ and $f \mapsto \sigma(f)$ are bijections, and in fact are inverse to each other.