

Algebra Quiz 3

Write your answers on the exam.

1. If G is a group and X is a G -set, what is the definition of an *orbit* of G on (or in) X ?

An orbit of G in X is a set of the form $\{gx : g \in G\}$ for some $x \in X$.

2. In the group S_5 consider the following equation:

$$\sigma(1\ 2\ 3\ 4\ 5)\sigma^{-1} = (3\ 4)(5\ 1\ 2)$$

- (a) Does there exist any σ which satisfies this condition? If not, explain why not.

No, because conjugate elements of S_n have cycle decompositions with corresponding lengths.

- (b) If so, give an example.

- (c) If so, say how many such σ exist. Explain your answer.

3. Answer the same questions for the equation:

$$\sigma(1\ 2)(3\ 4\ 5)\sigma^{-1} = (3\ 4)(5\ 1\ 2)$$

Yes, for example $\sigma = (1\ 3\ 5\ 2\ 4)$ will work. In general, the set of σ which will work is then a coset for the centralizer of $(1\ 2)(3\ 4\ 5)$, which is the product of the cyclic subgroups $\langle(1\ 2)\rangle$ and $\langle(3\ 4\ 5)\rangle$, which has order 6. Thus there are six such elements.