Solution to 4.3, #21: Suppose that M can be written in the form $\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}$, where A and C are square matrices. We claim that $\det(M) = \det(A)\det(C)$. First of all, if A or C is singular, then so is M, and both sides are zero. On the other hand, if A is nonsingular, then one can perform elementary row operators on A and on M without affecting C, and one finds that $\det(M) = \det(A)\det(M')$, where $M' = \begin{pmatrix} I & B' \\ 0 & C \end{pmatrix}$. Then one can do elementary row operations on C and M to find that $\det(M') = \det(C)\det(M'')$, where $M'' = \begin{pmatrix} I & B' \\ 0 & I \end{pmatrix}$. Finally, one can do elementary operations of the third kind only to find that $\det(M'') = I$.

Solution to 4.5, # 16:

Let $\delta\colon M_{n\times n} \to F$ be an alternating n-linear function. We claim that there is a $k \in F$ such that $\delta(A) = k\det(A)$ for all A. By Corollory 3 of 4.5, $\delta(EA) = \det(E)\delta(A)$ if E is any elementary matrix. Now if A is a product of elementary matrices, it follows by induction that $\delta(A) = \det(A)\delta(I)$. On the other hand, if A is singular, Corollary 2 says that $\delta(A) = 0$. Hence in any case, $\delta(A) = \det(A)k$ where $k := \det(I)$.