Assignment 1

MATH110

January 18, 2007

1.1 (a) $(3,1,2)$ and $(6,4,2)$ are not parallel because there is no constant c such that $3c = 6$, $c = 4$, and $2c = 2$.

(c) $(5,-6,7) = -1 \cdot (-5,6,-7)$ so the two vectors are parallel.

1.2 (c) $x = (3,7,2) + t(0,0,10)$

(d) $x = (-2,-1,5) + t(5,10,2)$

1.3 (b) $x = (3,-6,7) + t(5,-6,11) + s(-2,3,9)$

(c) $x = (-8,2,0) + t(-9,-1,0) + s(-14,7,0)$

1.4 The zero vector in the Euclidean plane is $(0,0)$. Given vector $\mathbf{v} = (x,y)$, we have

$$\mathbf{v} + (0,0) = (x,y) + (0,0) = (x+0, y+0) = (x,y) = \mathbf{v},$$

which is the desired property.

2.1 (a) True. Property 3 of vector spaces.

(b) False. Assume that there are two zero vectors, $\vec{0}$ and $\vec{0}'$. Then given any vector $\mathbf{x}$, $\mathbf{x}+\vec{0} = \mathbf{x} = \mathbf{x}+\vec{0}'$ by property of the zero vector. Then by the Cancellation Law for Vector Addition (page 11) $\vec{0} = \vec{0}'$.

(c) False. $\mathbf{x}$ could be the zero vector.

(d) False. a could be 0.

(e) True.

(f) False. An $m \times n$ matrix has m rows and n columns.

(g) False. Any two polynomials may be added.

(h) False. $f + g$ is a polynomial of degree less than or equal to n.

(i) True.

(j) True.

(k) True.

2.4 (a)

$$\begin{pmatrix} 2 & 5 & -3 \\ 1 & 0 & 7 \end{pmatrix} + \begin{pmatrix} 4 & -2 & 5 \\ -5 & 3 & 2 \end{pmatrix} = \begin{pmatrix} 6 & 3 & 2 \\ -4 & 3 & 9 \end{pmatrix}$$

(c)

$$4\begin{pmatrix} 2 & 5 & -3 \\ 1 & 0 & 7 \end{pmatrix} = \begin{pmatrix} 8 & 20 & -12 \\ 4 & 0 & 28 \end{pmatrix}$$

(e) $(2x^4 - 7x^3 + 4x + 3) + (8x^3 + 2x^2 - 6x + 7) = 2x^4 + x^3 + 2x^2 - 2x + 10$

2.7

$$\begin{array}{ll} f(0) = 2(0) + 1 = 1 & f(1) = 2(1) + 1 = 3 \\ g(0) = 1 + 4(0) - 2(0)^2 = 1 & g(1) = 1 + 4(1) - 2(1)^2 = 3 \\ h(0) = 5^0 + 1 = 2 & h(1) = 5^1 + 1 = 6 \end{array}$$

Therefore, $f = g$ and $f + g = h$ on the set $S = \{0,1\}$.